

Energy System Modelling

Summer Semester 2018, Lecture 2

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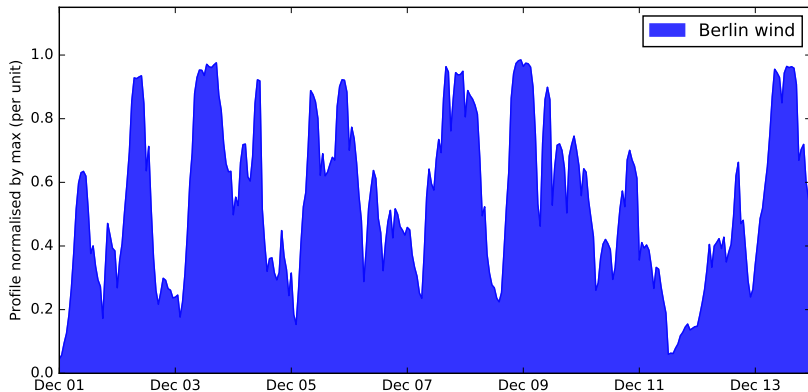
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Single location versus country versus
continent

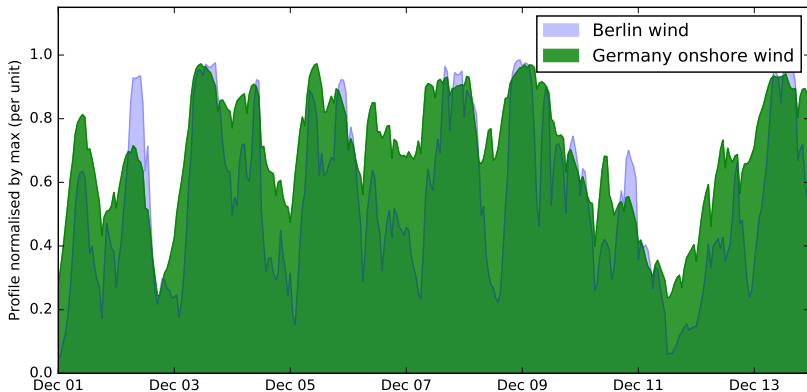
Variability: Single wind site in Berlin

Looking at the wind output of a single wind plant over two weeks, it is highly variable, frequently dropping close to zero and fluctuating strongly.



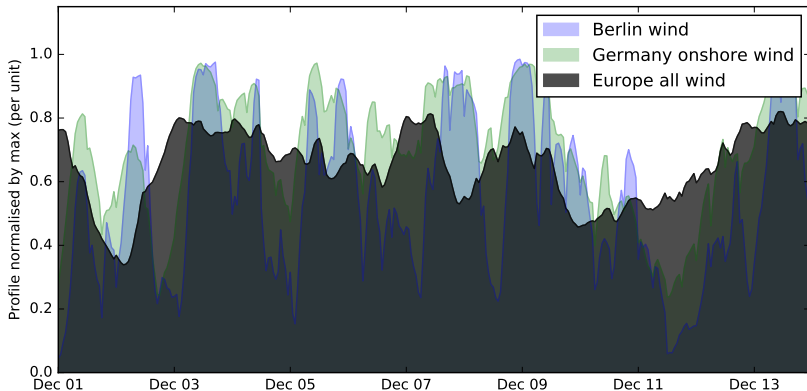
Variability: Single country: Germany

For a whole country like Germany this results in valleys and peaks that are somewhat smoother, but the profile still frequently drops close to zero.



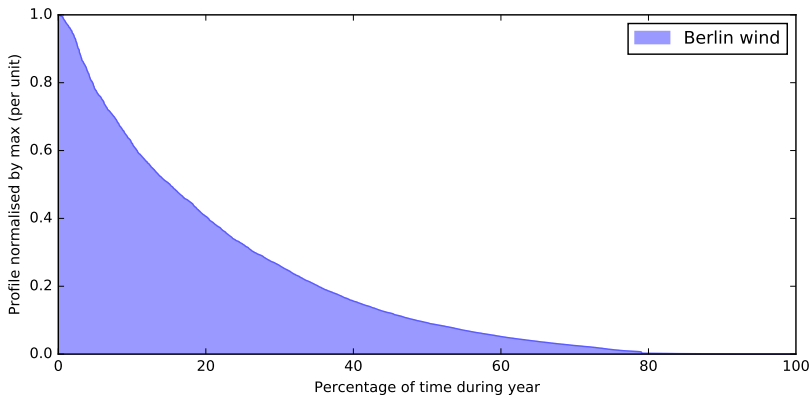
Variability: A continent: Europe

If we can integrate the feed-in of wind turbines across the European continent, the feed-in is considerably smoother: we've eliminated most valleys and peaks.



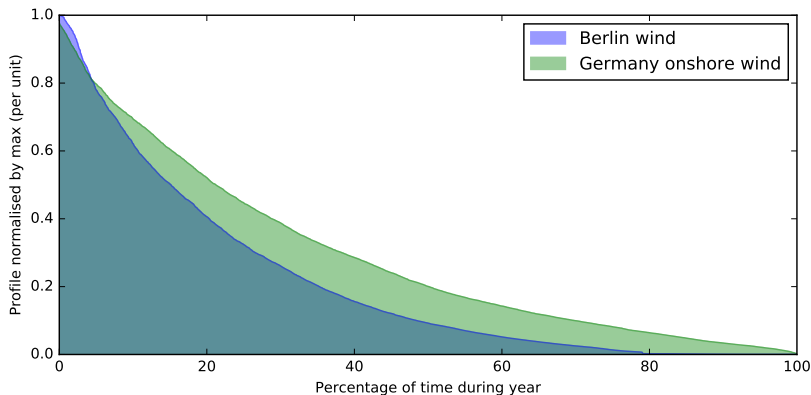
Duration curve: Berlin

A **duration curve** shows the feed-in for the whole year, re-ordered by from highest to lowest value. For a single location there are many hours with no feed-in.



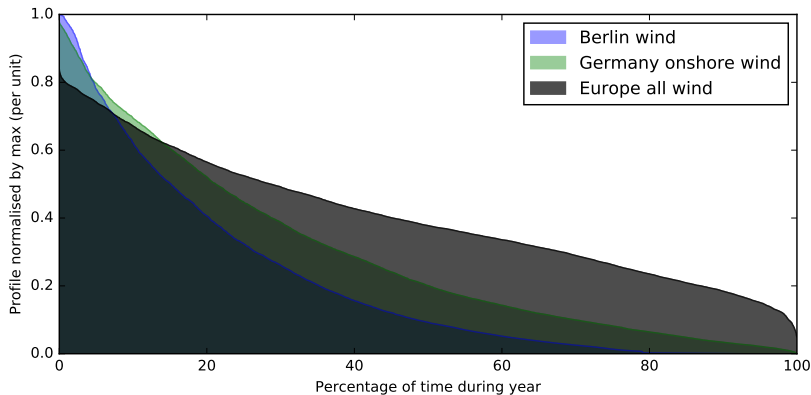
Duration curve: Germany

For a whole country there are fewer peaks and fewer hours with no feed-in.



Duration curve: Europe

For the whole of Europe there are no times with zero feed-in.



Statistical comparison

| Area | Mean | Standard deviation |
|-----------------------------|------|--------------------|
| Berlin | 0.21 | 0.26 |
| Germany | 0.26 | 0.24 |
| Europe (including offshore) | 0.36 | 0.19 |

Mismatch between load and renewables

How does the mismatch change as we integrate over larger areas?

If we have for each time t a demand of d_t and a 'per unit' availability w_t for wind and s_t for solar, then if we have W MW of wind and S MW of solar, the effective **residual load** or **mismatch** is

$$m_t = d_t - Ww_t - Ss_t$$

We choose W and S such that on **average** we cover all the load

$$\langle m_t \rangle = 0$$

and so that the 70% of the energy comes from wind and 30% from solar ($W = 147$ GW and $S = 135$ GW for Germany).

This means

$$W\langle w_t \rangle = 0.7\langle d_t \rangle$$

$$S\langle s_t \rangle = 0.3\langle d_t \rangle$$

Mismatch between load and renewables

Let p_t be the balance of power at each time. Because we cannot create or destroy energy, we need $p_t = 0$ at all times.

If the mismatch is positive $m_t > 0$, then we need **backup power** $b_t = m_t$ to cover the load in the absence of renewables, so that

$$p_t = b_t - m_t = b_t - d_t + Ww_t + Ss_t = 0$$

If the mismatch is negative $m_t < 0$ then we need **curtailment** $c_t = -m_t$ to reduce the excess feed-in from renewables, so that

$$p_t = -m_t - c_t = -c_t - d_t + Ww_t + Ss_t = 0$$

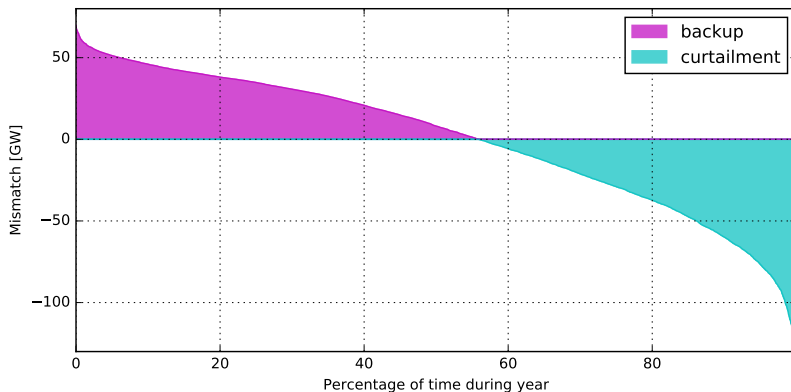
At any one time we have either backup or curtailment

$$p_t = b_t - m_t - c_t = Ww_t + Ss_t + b_t - d_t - c_t = 0$$

Mismatch for Germany

Backup generation needed for 31% of the total load.

Peak mismatch is 91% of peak load (around 80 GW).

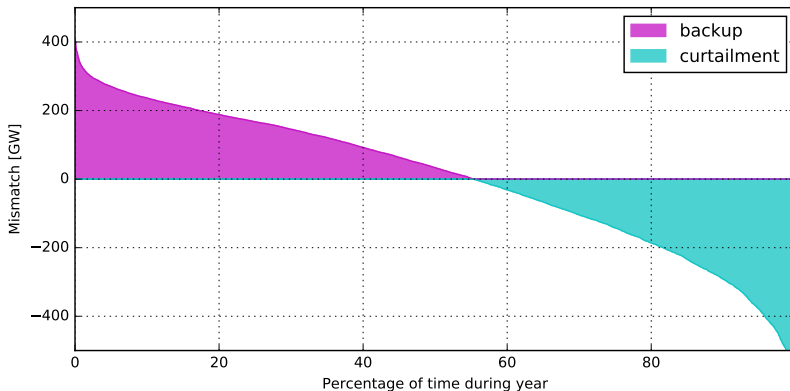


Mismatch for Europe

Requires 750 GW each of onshore wind and solar.

Backup generation needed for only 24% of the total load.

Peak mismatch is 79% of peak load (around 500 GW).

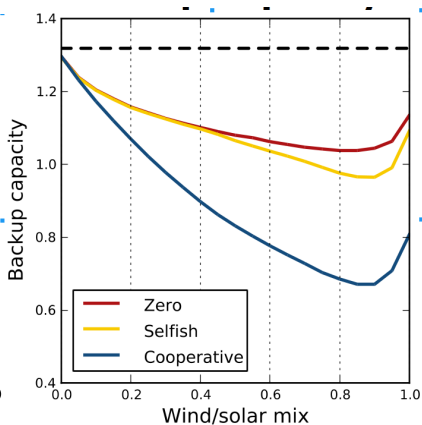
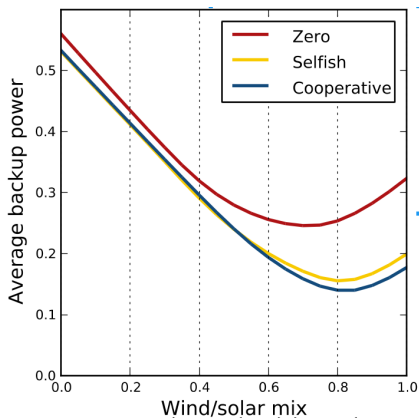


Conclusions

- Integration over a larger area smooths out the fluctuations of renewables, particularly wind
- Wind backs up wind
- This means we need **less backup energy**.
- and **less backup capacity**.

Greiner papers

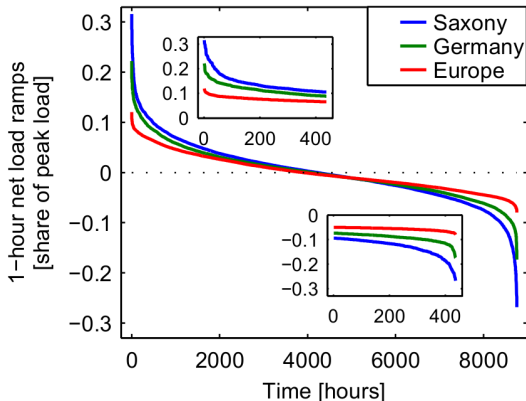
'Cost-optimal design of a simplified, highly renewable pan-European electricity system' by Rolando A. Rodriguez, Sarah Becker, Martin Greiner, Energy 83 (2015) 658-668



Flexibility Requirements

'Integration of wind and solar power in Europe: Assessment of flexibility requirements' by Huber, Dimkova, Hamacher, Energy 69 (2014) 236e246

1-hour net load ramp duration curves at the regional, country and European spatial scales at 50% share of renewables and 20% PV in the wind/PV mix for the meteorological year 2009.



Big Caveat

There is a big caveat to this analysis.

We've assumed that we can move power around Europe without penalty.

However, in reality, we can only transport within restrictions of the power network.

In general we will have different power imbalances $p_{i,t}$ at each location/node i and instead of $p_t = 0$ we will have

$$\sum_i p_{i,t} = 0$$

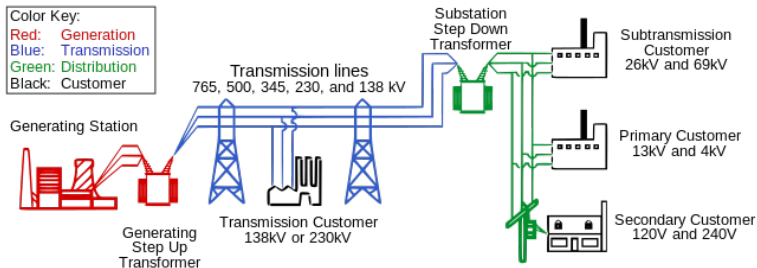
(neglecting power losses in the network).

Moving excess power to locations of consumption is the role of the network.

Networks

Electricity Transport from Generators to Consumers

Electricity is also easy to transport over long distances using the high voltage transmission grid:



Usually in houses the voltage is 230 V, but in the transmission grid it is hundreds of thousands of Volts.

Source: Wikipedia

European transmission network

Flows in the European transmission network must respect both Kirchoff's laws for physical flow and the thermal limits of the power lines.

Taking account of network flows and constraints in the electricity market is a major and exciting topic at the moment.

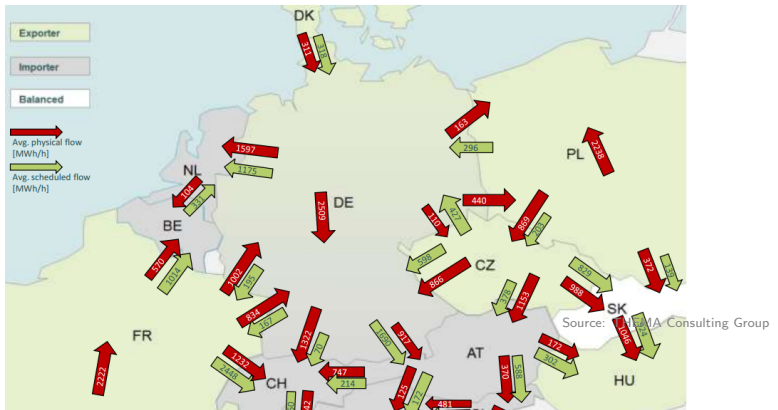


Source: ENTSO-E

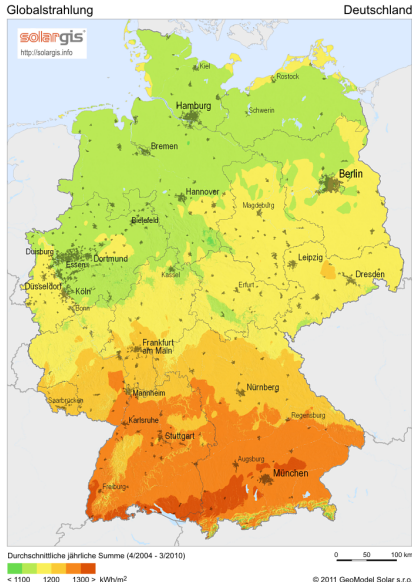
Network Bottlenecks and Loop Flows

Electricity is traded in large market zones. Power trades between zones (“scheduled flows”) do not always correspond to what flows according to the network physics (“physical flows”). This leads to political tension as wind from Northern Germany flows to Southern Germany via Poland and the Czech Republic.

Figure 7: Average physical and scheduled flows [MWh/h], 01.01.2011 – 31.12.2012

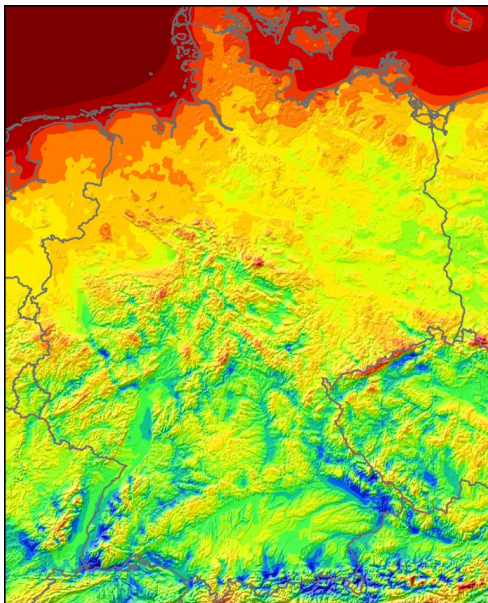


Solar resource distribution in Germany

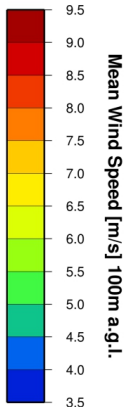


- Solar insolation at top of atmosphere is on average 1361 W/m^2 (orbit is elliptical).
- In Germany average insolation on a horizontal surface is around 1200 kWh/m^2 .
- A 1 kW solar panel (around 7 m^2) will generate around 1000 kWh/a .

Wind resource distribution in Germany



2004 - 2013

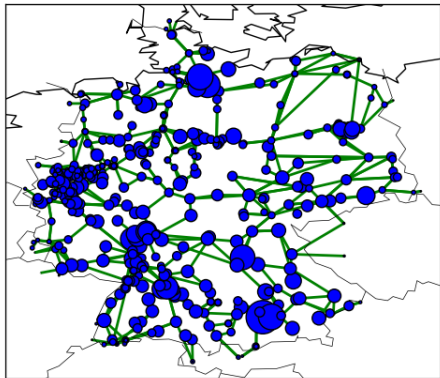


- Best wind speeds in Germany in North and on hills.
- In theory power output goes like cube $\propto v^3$ of wind speed v .
- In practice power-speed relationship is only partially cubic.

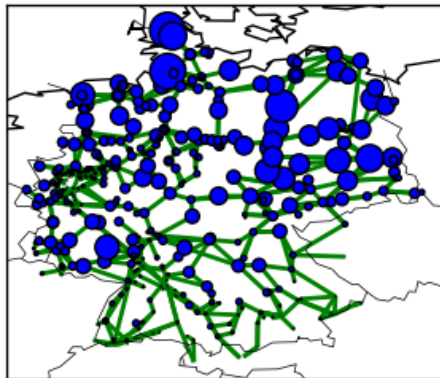
The Problem

Renewables are not always located near demand centres, as in this example from Germany.

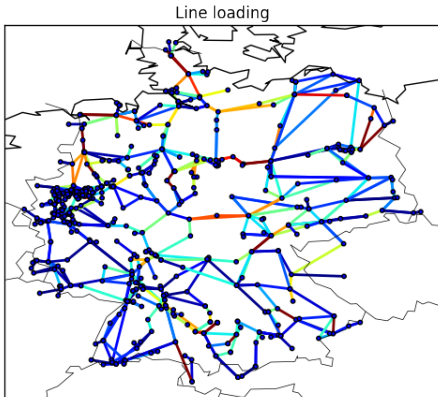
Load distribution



Wind Onshore



The Problem



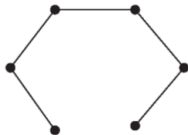
- This leads to **overloaded lines** in the middle of Germany, which cannot transport all the wind energy from North Germany to the load in South Germany
- It also overloads lines in neighbouring countries due to **loop flows** (unplanned physical flows 'according to least resistance' which do not correspond to traded flows)
- It also **blocks imports and exports** with neighbouring countries, e.g. Denmark

Different types of networks: radial networks

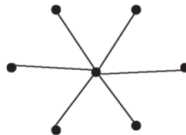
In a **radial** network there is only one path between any two nodes on the network.

The power flow is a simple function of the nodal power imbalances.

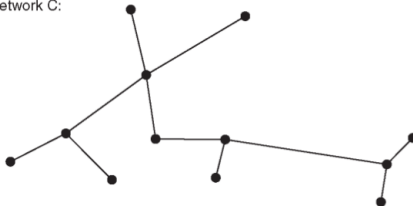
Network A:



Network B:



Network C:



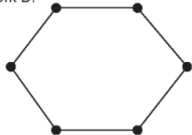
Source: Biggar & Hesamzadeh

Different types of networks: meshed networks

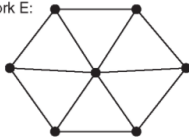
In a **meshed** network there are at least two nodes with multiple paths between them.

The power flow is now a function of the impedances in the network.

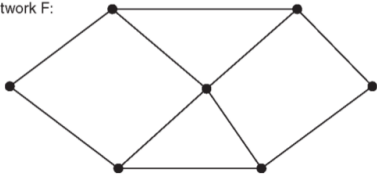
Network D:



Network E:



Network F:



Source: Biggar & Hesamzadeh

Graph Theory

Definition of a network

Our definition (Newman): A network (graph) is a collection of vertices (nodes) joined by edges (links).

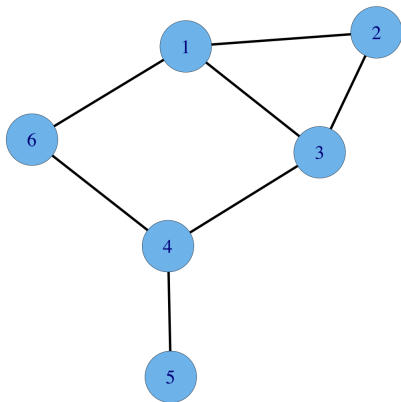
More precise definition (Bollobàs): A graph G is an ordered pair of disjoint sets (V, E) such that E (the edges) is a subset of the set $V^{(2)}$ of unordered pairs of V (the vertices).

Edge list representation

- Vertices:
1,2,3,4,5,6
- Edges:
(1,2), (1,3), (1,6), (2,3),
(3,4), (4,5), (4,6)

Definition from graph theory:

- $n = 6$ vertices: *order* of the graph
- $m = 7$ edges: *size* of the graph

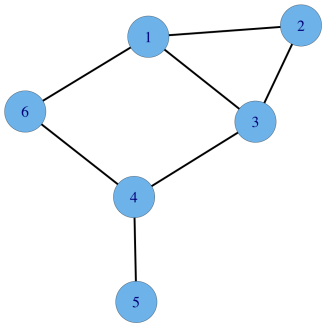


Adjacency matrix **A**

$$A_{ij} = \begin{cases} 1 & \text{if there is an edge between vertices } i \text{ and } j \\ 0 & \text{otherwise.} \end{cases}$$

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

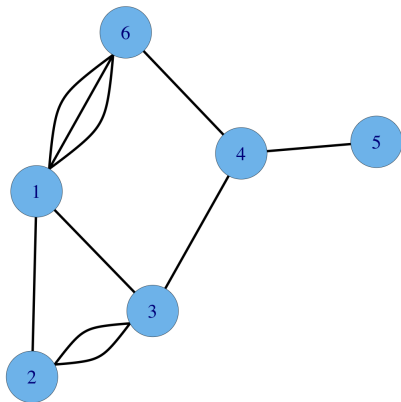
- Diagonal elements are zero.
- Symmetric matrix.
- If there are N vertices, it's an $N \times N$ matrix.



Multigraph

There can be more than one edge between a pair of vertices.

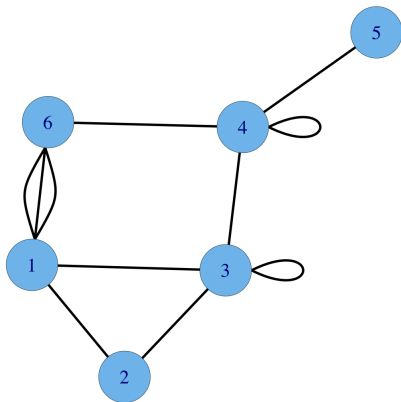
$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 3 \\ 1 & 0 & 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$



Self-edges

There can be self-edges (also called self-loops).

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 3 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

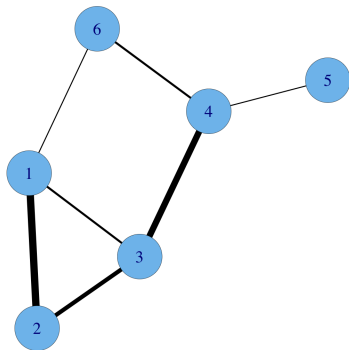


- Diagonal elements can be non-zero:
Definition: $A_{ii} = 2$ for one self-edge.

Weighted networks

Weight or *strength* assigned to each edge.

$$\mathbf{A} = \begin{pmatrix} 0 & 1.4 & 0.4 & 0 & 0 & 0.8 \\ 1.4 & 0 & 1.2 & 0 & 0 & 0 \\ 0.4 & 1.2 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0.2 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.2 & 0 & 0 \\ 0.8 & 0 & 0 & 0.4 & 0 & 0 \end{pmatrix}$$



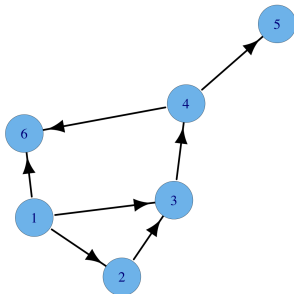
Weights can be both positive or negative.

Directed Networks (Digraphs)

Edge is pointing from one vertex to another (*directed edge*).

$$A_{ij} = \begin{cases} 1 & \text{if there is an edge from } j \text{ to } i \\ 0 & \text{otherwise.} \end{cases}$$

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$



In general the adjacency matrix of a directed network is asymmetric.

Degree

- Degree k_i of vertex i : Number of edges connected to i .
- Average degree of the network: $\langle k \rangle$.

In terms of the adjacency matrix \mathbf{A} :

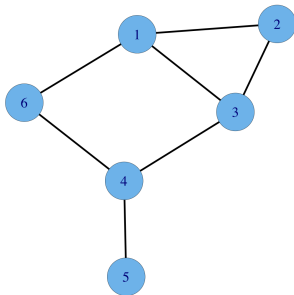
$$k_i = \sum_{j=1}^n A_{ij} \quad , \quad \langle k \rangle = \frac{1}{n} \sum_i k_i = \frac{1}{n} \sum_{i=1}^n \sum_{j=1}^n A_{ij} .$$

$$k_5 = 1$$

$$k_2 = k_6 = 2$$

$$k_1 = k_3 = k_4 = 3$$

$$\langle k \rangle = 2.33$$



Examples

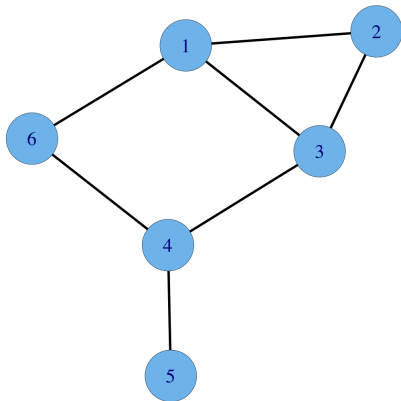
| NETWORK | NODES | LINKS | DIRECTED UNDIRECTED | N | L | $\langle k \rangle$ |
|-----------------------|----------------------------|----------------------|------------------------|---------|------------|---------------------|
| Internet | Routers | Internet connections | Undirected | 192,244 | 609,066 | 6.34 |
| WWW | Webpages | Links | Directed | 325,729 | 1,497,134 | 4.60 |
| Power Grid | Power plants, transformers | Cables | Undirected | 4,941 | 6,594 | 2.67 |
| Mobile Phone Calls | Subscribers | Calls | Directed | 36,595 | 91,826 | 2.51 |
| Email | Email addresses | Emails | Directed | 57,194 | 103,731 | 1.81 |
| Science Collaboration | Scientists | Co-authorship | Undirected | 23,133 | 93,439 | 8.08 |
| Actor Network | Actors | Co-acting | Undirected | 702,388 | 29,397,908 | 83.71 |
| Citation Network | Paper | Citations | Directed | 449,673 | 4,689,479 | 10.43 |
| E. Coli Metabolism | Metabolites | Chemical reactions | Directed | 1,039 | 5,802 | 5.58 |
| Protein Interactions | Proteins | Binding interactions | Undirected | 2,018 | 2,930 | 2.90 |

(from the free textbook "Network Science")

Degree matrix **D**

$$D_{ij} = \begin{cases} k_i & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases}$$

$$\mathbf{D} = \begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

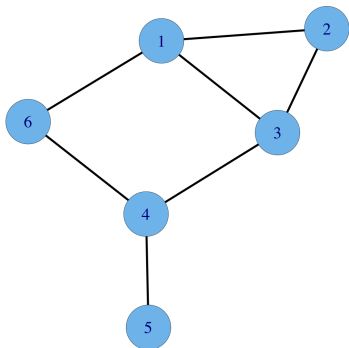


Laplacian

$$\mathbf{L} = \mathbf{D} - \mathbf{A}$$

$$\mathbf{L} = \begin{pmatrix} 3 & -1 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ -1 & -1 & 3 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & -1 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & -1 & 0 & 2 \end{pmatrix}$$

- \mathbf{L} inherits symmetry from \mathbf{D} and \mathbf{A} .
- The number of zero eigenvalues equals the number of connected components.



Why is it called the Laplacian?

What does this matrix have to do with the second-order Laplacian:

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad (1)$$

we know and love from continuous physics?

On a 1d lattice, for each link (difference) from K^t get $u_i - u_{i-1} \sim \frac{d}{dx}$.

From $L = KK^t$ get $2u_i - u_{i-1} - u_{i+1} \sim \frac{d^2}{dx^2}$.

Similarly for 2d lattice, from the Laplacian you get

$$4u_{i,j} - u_{i+1,j} - u_{i-1,j} - u_{i,j+1} - u_{i,j-1} \sim \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad (2)$$

which is a second-order difference in both x and y directions.

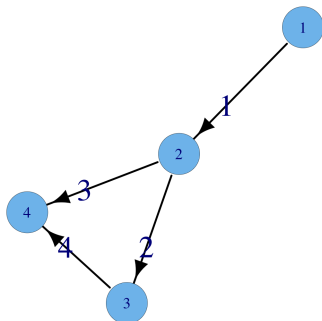
In fact you can do interesting discrete physics with these matrices (more later...).

The incidence matrix

For a directed graph (every edge has an orientation) $G = (V, E)$ with N nodes and L edges, the node-edge **incidence matrix** $K \in \mathbb{R}^{N \times L}$ has components

$$K_{i\ell} = \begin{cases} 1 & \text{if edge } \ell \text{ starts at node } i \\ -1 & \text{if edge } \ell \text{ ends at node } i \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{K} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix}$$



Incidence matrix properties

The incidence matrix has several important properties.

First, for a given edge ℓ , $\sum_i K_{i\ell} = 0$, i.e. every edge starts at some node (+1) and ends at some node (-1).

It is related to the Laplacian matrix by

$$L = KK^t$$

Check the definitions agree:

$$L_{ij} = \sum_{\ell} K_{i\ell} K_{j\ell}$$

for $i = j$ and $i \neq j$.

The kernel of the incidence matrix

The kernel of K_{il} , i.e. particular combinations of edges which are annihilated by K , has a very special meaning.

Consider the combination of edges $(0, 1, -1, 1)^t$

$$\mathbf{K} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

This corresponds to a **closed cycle** in the graph.

The matrix K can be interpreted as a **boundary operator**. A cycle has no boundary in 0-d. There is a general theory called **homology theory**, which can compute topological invariants of manifolds called **homology groups**.

Cycle matrix

We can organise the cycles in a matrix C_{ℓ_C} , where c labels each cycle.

We have

$$KC = 0$$

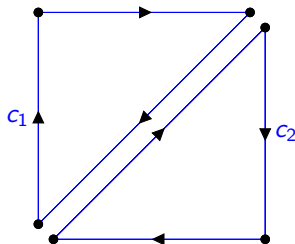
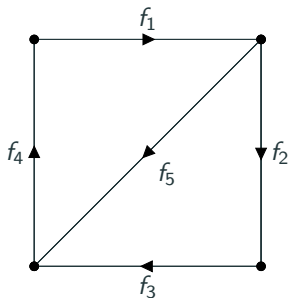
by definition of C being in the kernel.

The image of K has dimension $N - 1$ (i.e. the rank of K), so the number of cycles (i.e. the nullity of K) is $L - N + 1$.

In our case $L = 4$, $N = 4$ so there is only 1 cycle

$$\mathbf{C} = (0, 1, -1, 1)^t$$

Independent basis of cycles



Two independent cycles:

$$c_1 = f_1 + f_5 + f_4$$

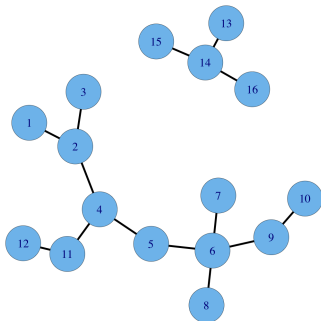
$$c_2 = f_2 + f_3 + -f_5$$

The outer cycle is not independent:

$$c_3 = f_1 + f_2 + f_3 + f_4 = c_1 + c_2$$

Trees

- A collection of trees is called a *forest*.
- Trees play an import role for random graph models.
- In a tree, there is exactly one path between any pair of vertices.
- A tree of N vertices always has exactly $N - 1$ edges.
- Any connected network with N vertices and $N - 1$ edges is a tree.
- Trees have **no cycles**.

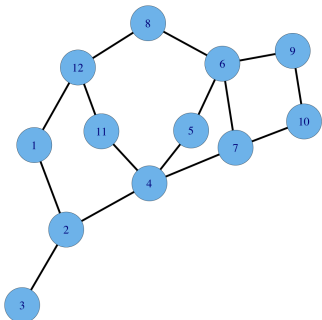


Planar networks

A *planar network* is a network that can be drawn on a plane without having any edges cross.

Examples:

- Trees
- Road networks (approximately)
- Power grids (approximately)
- Shared borders between countries, etc.



Paths

- Route through the network, from vertex to vertex along the edges
- Defined for both directed and undirected networks
- Special case: self-avoiding paths
- *Length* of a path: number of edges along the path ("hops")
- Number of paths of length r between vertices i and j :

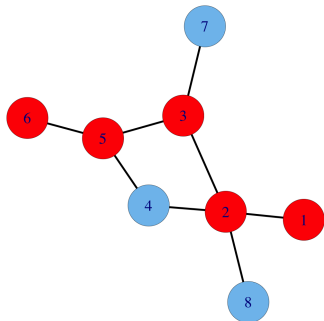
$$N_{ij}^{(r)} = [\mathbf{A}^r]_{ij}$$

- Total number L_r of loops of length r anywhere in the network:

$$L_r = \sum_{i=1}^n [\mathbf{A}^r]_{ii} = \text{Tr} \mathbf{A}^r .$$

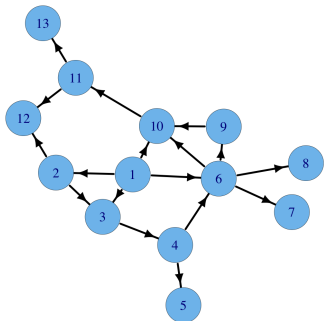
Geodesic / shortest paths

- A path between two vertices such that no shorter path exists
- Geodesic distance between vertices i and j is the smallest value of r such that $[\mathbf{A}^r]_{ij} > 0$.
- Self-avoiding
- In general not unique
- *Diameter* of a network: Length of the longest geodesic path between any pair of vertices



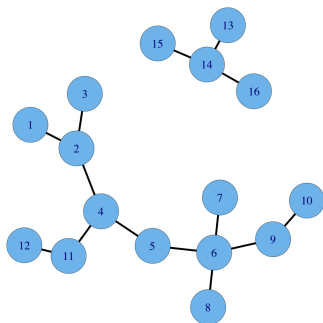
Acyclic directed network

- Directed network without closed loops of edges (DAG)
- Examples: power flow in an electricity grid, citation network of papers
- Topological ordering: For every directed edge $i \rightarrow j$, vertex i comes before j in the ordering:
(1,2,3,4,6,9,10,11,12,8,7,5,13)
- With a topological ordering, the adjacency matrix of an acyclic directed network is *strictly triangular*



Components of networks

- Subgroups of vertices with no connections between the respective groups
- *Disconnected* network
- Subgroups: *components*
- Adjacency matrix: Block-diagonal form



(Co)homology analogy

$$K \leftrightarrow \delta \text{ (1d boundary operator)}$$

$$K^t \leftrightarrow d \text{ (0d differential)}$$

$$L = KK^t \leftrightarrow \Delta = d * d \text{ (0d Laplacian)}$$

On a 1d lattice, for each link (difference) from K^t get $u_i - u_{i-1} \sim \frac{d}{dx}$.

From $L = KK^t$ get $2u_i - u_{i-1} - u_{i+1} \sim \frac{d^2}{dx^2}$.

Similarly for 2d lattice.

Computing the Linear Power Flow

The goal of power flow analysis

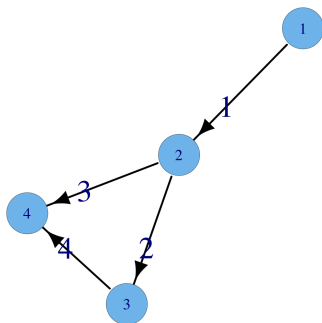
The goal of a power/load flow analysis is to find the flows in the lines of a network given a power injection pattern at the nodes.

I.e. given power injection at the nodes

$$\mathbf{P}_i = \begin{pmatrix} 50 \\ 50 \\ 0 \\ -100 \end{pmatrix}$$

what are the flows in lines 1-4?

To find the flows, it is sufficient to know the **impedances** of the lines and the **voltages** at each node.



Framing the load flow problem

Suppose we have N nodes labelled by i , and L edges labelled by ℓ forming a directed graph G .

Suppose at each node we have a **power imbalance** p_i ($p_i > 0$ means its generating more than it consumes and $p_i < 0$ means it is consuming more than it).

Since we cannot create or destroy energy (and we're ignoring losses):

$$\sum_i p_i = 0$$

Question: How do the flows f_ℓ in the network relate to the nodal power imbalances?

Answer: According to the impedances (generalisation of resistance for oscillating voltage/current) and the corresponding voltages.

Kirchhoff's Current Law (KCL)

KCL says (in this linear setting) that the nodal power imbalance at node i is equal to the sum of direct flows arriving at the node. This can be expressed compactly with the incidence matrix

$$p_i = \sum_{\ell} K_{i\ell} f_{\ell} \quad \forall i$$

Kirchhoff's Voltage Law (KVL)

KVL says that the sum of voltage differences across edges for any closed cycle must add up to zero.

If the voltage at any node is given by θ_i (this is infact the voltage **angle** - more next week) then the voltage difference across edge ℓ is

$$\sum_i K_{i\ell} \theta_i$$

And Kirchhoff's law can be expressed using the cycle matrix encoding of independent cycles

$$\sum_{\ell} C_{\ell c} \sum_i K_{i\ell} \theta_i = 0 \quad \forall c$$

[Automatic, since we already said $KC = 0$.]

Kirchhoff's Voltage Law (KVL)

If we express the flow on each line in terms of the voltage angle (a relative of $V = IR$) then for a line ℓ with reactance x_ℓ

$$f_\ell = \frac{\theta_i - \theta_j}{x_\ell} = \frac{1}{x_\ell} \sum_i K_{i\ell} \theta_i$$

KVL now becomes

$$\sum_\ell C_{\ell c} x_\ell f_\ell = 0 \quad \forall c$$

Solving the equations

If we combine

$$f_\ell = \frac{1}{x_\ell} \sum_i K_{i\ell} \theta_i$$

with Kirchhoff's Current Law we get

$$p_i = \sum_\ell K_{i\ell} f_\ell = \sum_\ell K_{i\ell} \frac{1}{x_\ell} \sum_j K_{j\ell} \theta_j$$

This is a **weighted Laplacian**. If we write $B_{k\ell}$ for the diagonal matrix with $B_{\ell\ell} = \frac{1}{x_\ell}$ then

$$L = KBK^t$$

and we get a **discrete Poisson equation** for the θ_i sourced by the p_i

$$p_i = \sum_j L_{ij} \theta_j$$

We can solve this for the θ_i and thus find the flows.