


Energy Systems, Summer Semester 2026

Lecture 3: Electricity Time Series

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1. Electricity Consumption
2. Electricity Generation
3. Variable Renewable Energy (VRE)
4. Balancing a Single Country

Electricity Consumption

Electricity is a versatile, high-energy form of energy carried by electrical charge which can be consumed in a wide variety of ways (with selected examples):

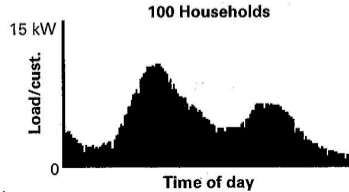
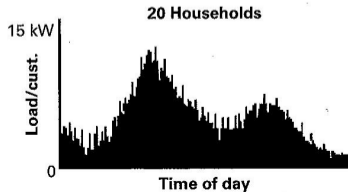
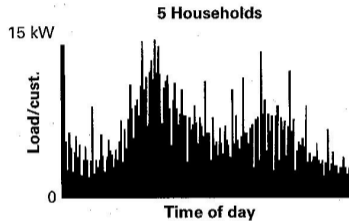
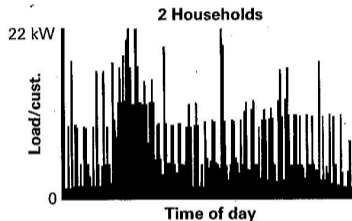
- Lighting (lightbulbs, halogen lamps, televisions)
- Mechanical work (hoovers, washing machines, electric vehicles)
- Heating (cooking, resistive room heating, heat pumps)
- Cooling (refrigerators, air conditioning)
- Electronics (computation, data storage, control systems)
- Industry (electrochemical processes)

Compare the convenience and versatility of electricity with another energy carrier: the chemical energy stored in coal, which can only be accessed by burning it.

At full power, the following items consume:

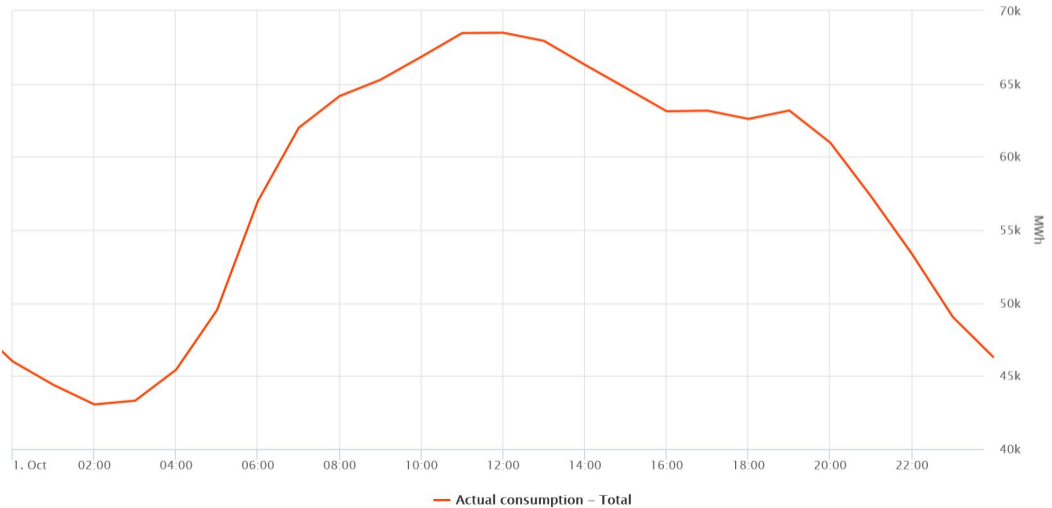
Item	Power
New efficient lightbulb	10 W
Old-fashioned lightbulb	70 W
Single room air-conditioning	1.5 kW
Kettle	2 kW
Factory	~1-500 MW
CERN	200 MW
Germany total demand	35-80 GW

The discrete actions of individual consumers smooth out statistically if we aggregate over many consumers.



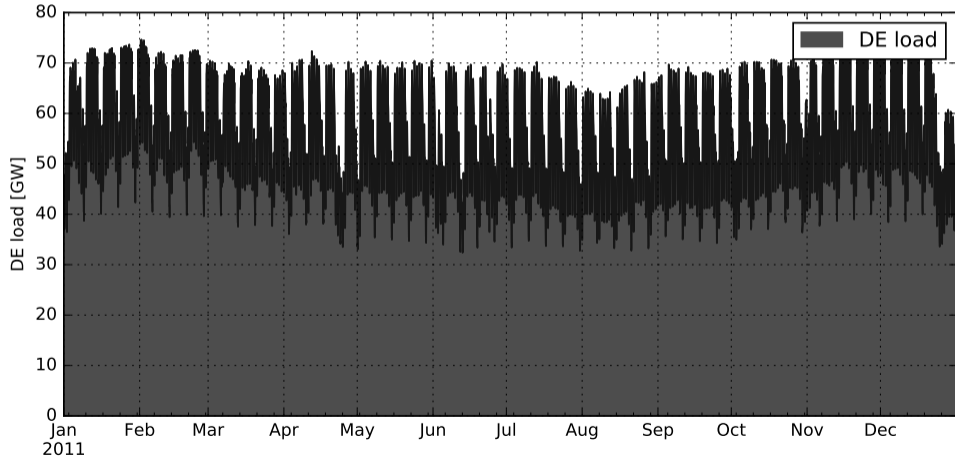
National daily load curve

The national German load curve includes residential, services and industry demand.

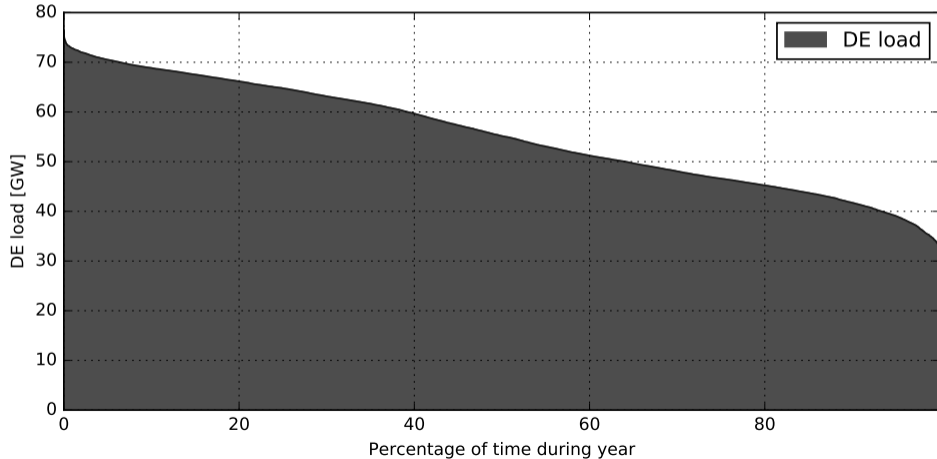


Load curve properties

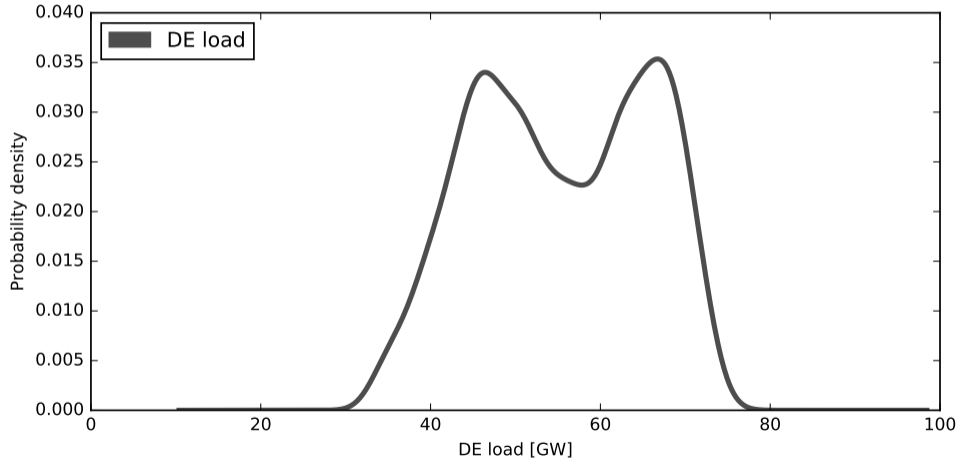
The Germany load curve (around 500 TWh/a) shows **daily**, **weekly** and **seasonal** patterns; religious festivals are also visible.

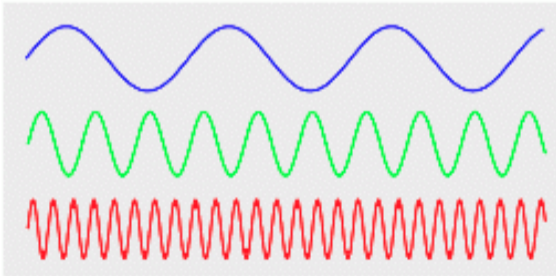
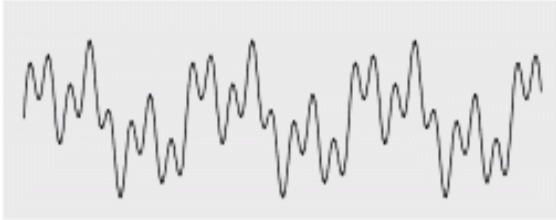


For some analysis it is useful to construct a **duration curve** by stacking the hourly values from highest to lowest.



Similarly we can also build the **probability density function** $pdf(x)$, $\int dx pdf(x) = 1$:



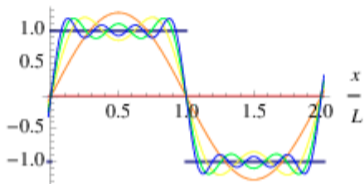


- Fourier analysis decomposes a **periodic** signal into simpler sine waves
- Every periodic signal can be broken down into a sum of sine waves with different **frequencies**

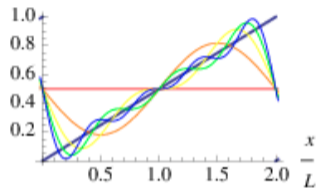
Fourier transform to see spectrum of periodic signals

Common examples of Fourier approximations using more and more terms with high frequencies:

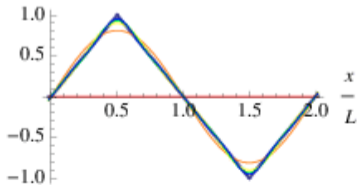
square wave



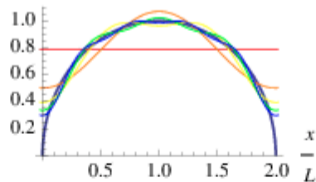
sawtooth wave



triangle wave



semicircle



For a periodic, continuous, complex signal $f(t)$, we can decompose it in frequency space to see which frequencies dominate the signal. This is called a **Fourier transform/series**.

For period T (in our case a year) the function $f : [0, T] \rightarrow \mathbb{C}$ can be decomposed

$$f(t) = \sum_{n=-\infty}^{n=\infty} a_n e^{-\frac{i2\pi nt}{T}}$$

To recover the values of the **frequency amplitudes** a_n , integrate over T

$$a_n = \frac{1}{T} \int_0^T dt \left[f(t) e^{\frac{i2\pi nt}{T}} \right]$$

For a real-valued function $f : [0, T] \rightarrow \mathbb{R}$, $a_{-n} = a_n^*$.

For a periodic, **discrete** signal f_n , the **Fast Fourier Transform** (FFT) is a computationally advantageous algorithm and is implemented in many programming libraries (see tutorial).

To remind yourself of how Fourier transforms work, check the formula for a_n by inserting the expansion of $f(t)$ into the formula for a_n .

First hint: remember Euler's formula:

$$e^{i\theta} = \cos \theta + i \sin \theta$$

Second hint: think about integrating a periodic signal over its period:

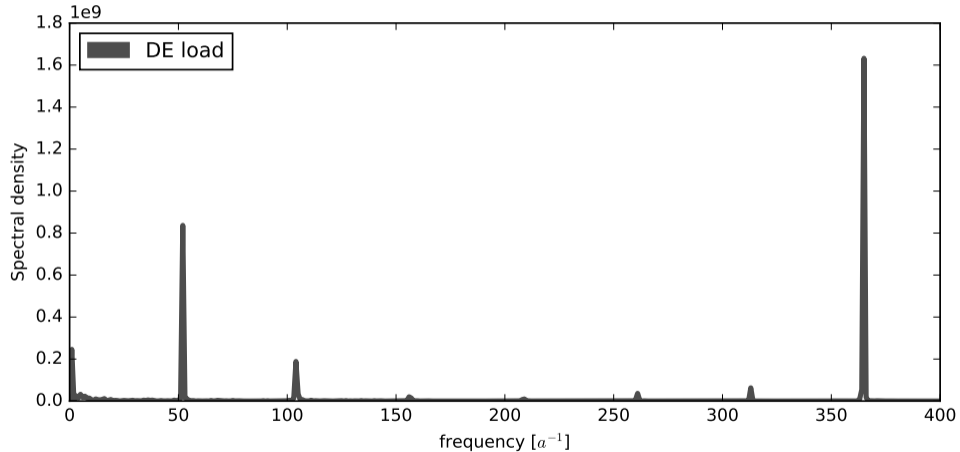
$$\frac{1}{T} \int_0^T dt \cos \frac{2\pi nt}{T} = \begin{cases} 1, & \text{if } n = 0 \\ 0, & \text{otherwise} \end{cases}$$

Inserting the expansion of $f(t)$ into the formula for a_n :

$$\begin{aligned}\frac{1}{T} \int_0^T dt \left[f(t) e^{\frac{i2\pi nt}{T}} \right] &= \frac{1}{T} \int_0^T dt \left[\sum_{m=-\infty}^{m=\infty} a_m e^{-\frac{i2\pi mt}{T}} e^{\frac{i2\pi nt}{T}} \right] \\ &= \frac{1}{T} \sum_{m=-\infty}^{m=\infty} a_m \int_0^T dt \left[e^{\frac{i2\pi(n-m)t}{T}} \right] \\ &= \frac{1}{T} \sum_{m=-\infty}^{m=\infty} a_m \int_0^T dt \left[\cos\left(\frac{2\pi(n-m)t}{T}\right) + i \sin\left(\frac{2\pi(n-m)t}{T}\right) \right] \\ &= a_n\end{aligned}$$

Since integrating sine over its period gives zero, and cosine is only zero for $n - m = 0$.

If we Fourier transform, the **seasonal**, **weekly** and **daily** frequencies are clearly visible.



Electricity Generation

Conservation of Energy: Energy cannot be created or destroyed: it can only be converted from one form to another.

There are several 'primary' sources of energy which are converted into electrical energy in modern power systems:

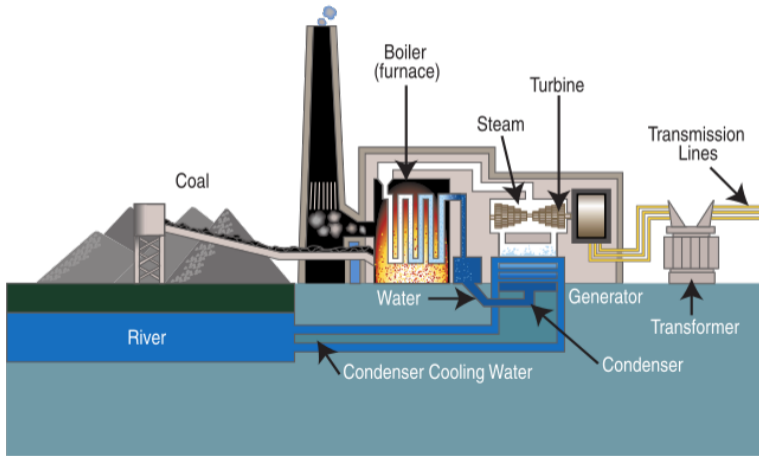
- Chemical energy, accessed by combustion (coal, gas, oil, biomass)
- Nuclear energy, accessed by fission reactions, perhaps one day by fusion too
- Hydroelectric energy, allowing water to flow downhill (gravitational potential energy)
- Wind energy (kinetic energy of air)
- Solar energy (accessed with photovoltaic (PV) panels or concentrating solar thermal power (CSP))
- Geothermal energy

NB: The definition of 'primary' is somewhat arbitrary.

At full power, the following items generate:

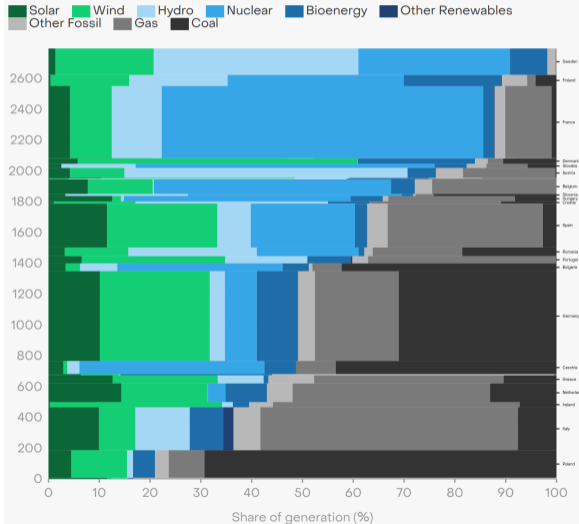
Item	Power
Solar panel on house roof	15 kW
Wind turbine	3 MW
Coal power station	1 GW

With the exception of solar photovoltaic panels (and electrochemical energy and a few other minor exceptions), all generators convert to electrical energy via rotational kinetic energy and electromagnetic induction in an *alternating current generator*.

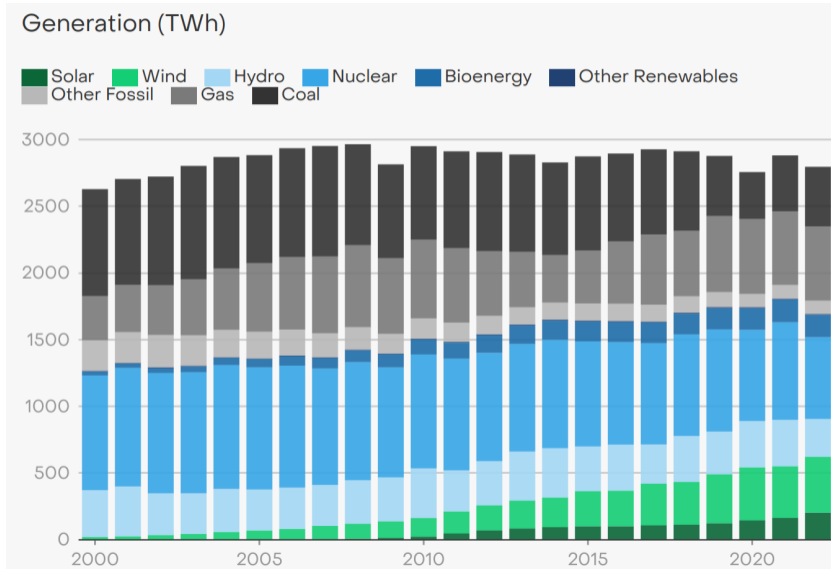


Electricity generation in EU countries in 2022

Electricity generation (TWh, y-axis) and share of electricity (% , x-axis)



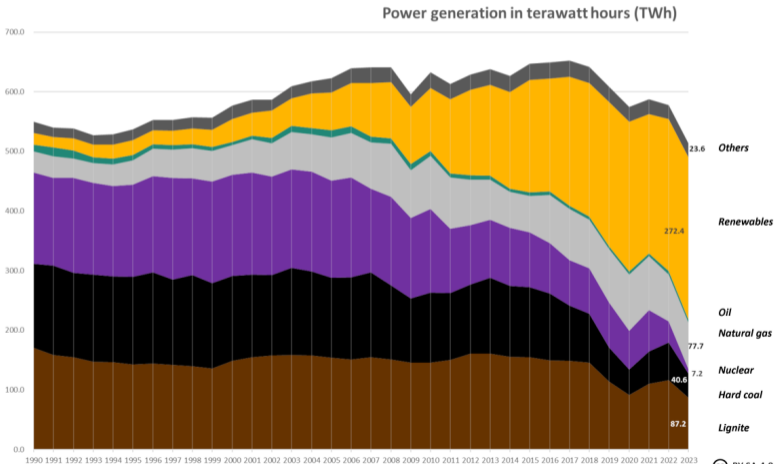
39% (1,104 TWh) of the EU's electricity is generated from coal, gas and other fossil sources. Coal produces 16% (447 TWh), gas 20% (557 TWh) and other fossil fuels 3.6% (100 TWh). Nuclear remains the single largest contributor to EU electricity at 22% (613 TWh) of the mix. 15% (420 TWh) is produced by wind and 7.3% (203 TWh) is produced by solar. Combined, wind and solar produce more electricity than any other fuel (22%, 623 TWh). The rest is produced by hydro (10%, 283 TWh), bioenergy (6%, 167 TWh) and other renewables (0.2%, 6.7 TWh).



Renewables reached 52% of gross electricity in Germany in 2023

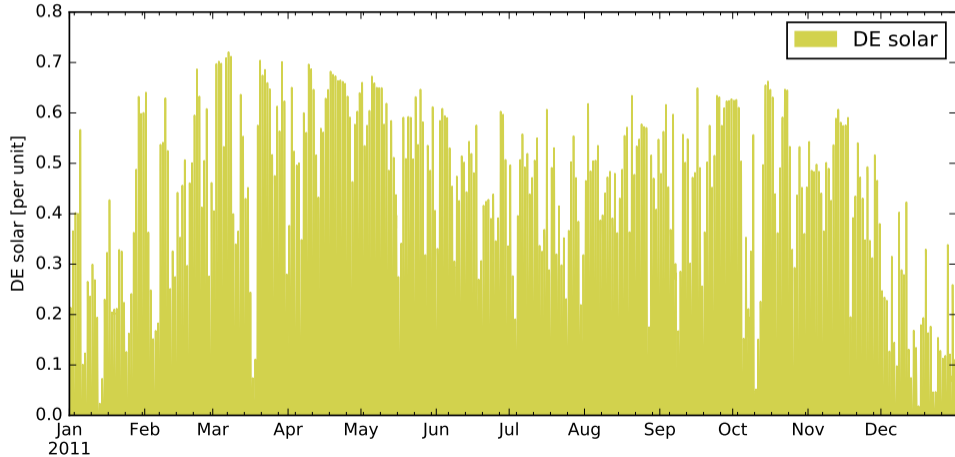
Gross power production in Germany 1990 - 2023, by source.

Data: AGEB 2024.



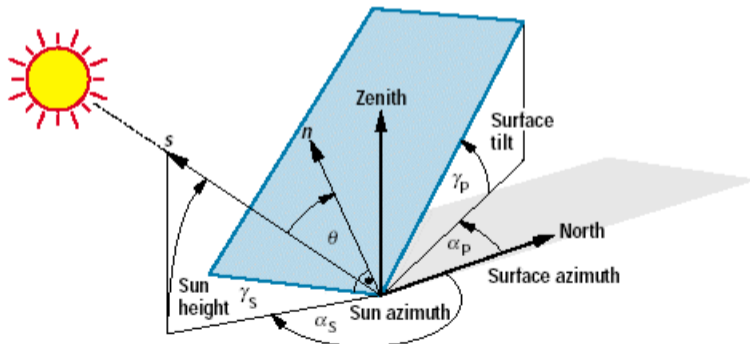
Variable Renewable Energy (VRE)

Unlike the load, the solar feed-in is much more variable, dropping to zero and not reaching full output (when aggregated over all of Germany).

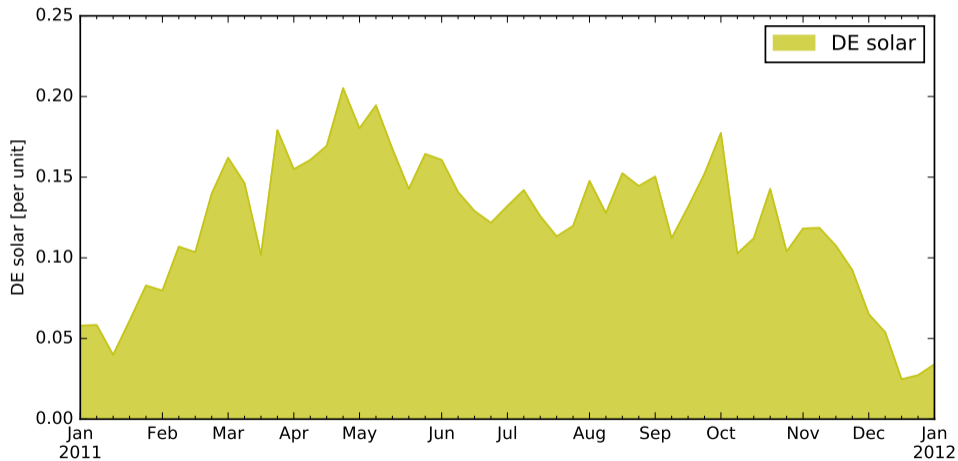


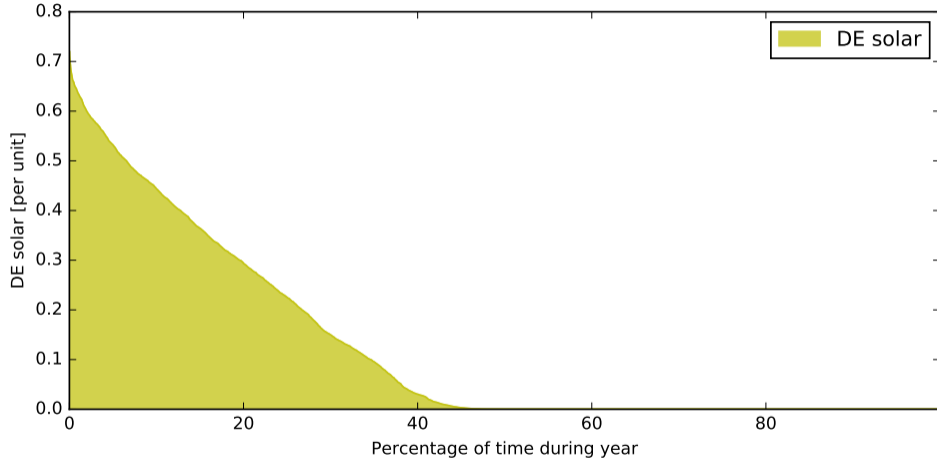
How do we derive solar time series?

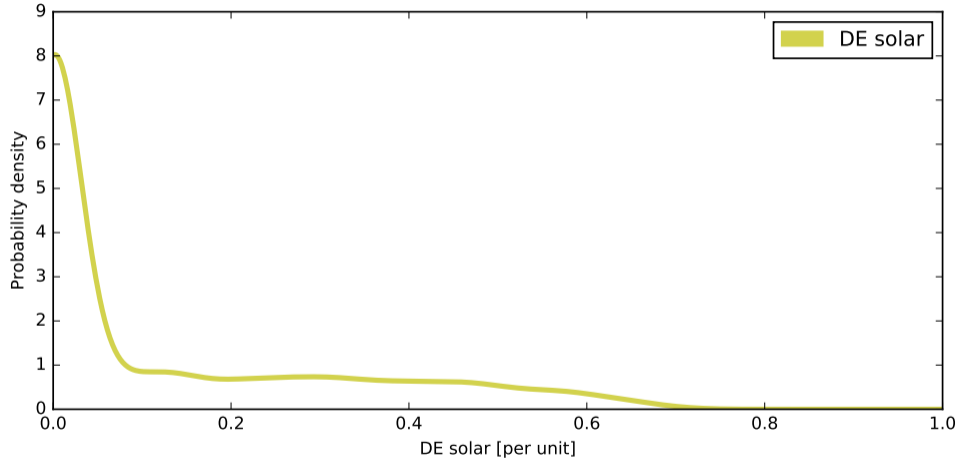
We take times series weather data for the solar radiation (also called irradiation or insolation) at each location in W/m^2 . This is often provided for a horizontal surface, so we need to convert for the angles of the solar panel to the horizontal, and account for factors that affect the energy conversion (losses, outside temperature). We have a software library **atlite** that takes care of this. See <https://model.energy> or <https://renewables.ninja> for live examples.



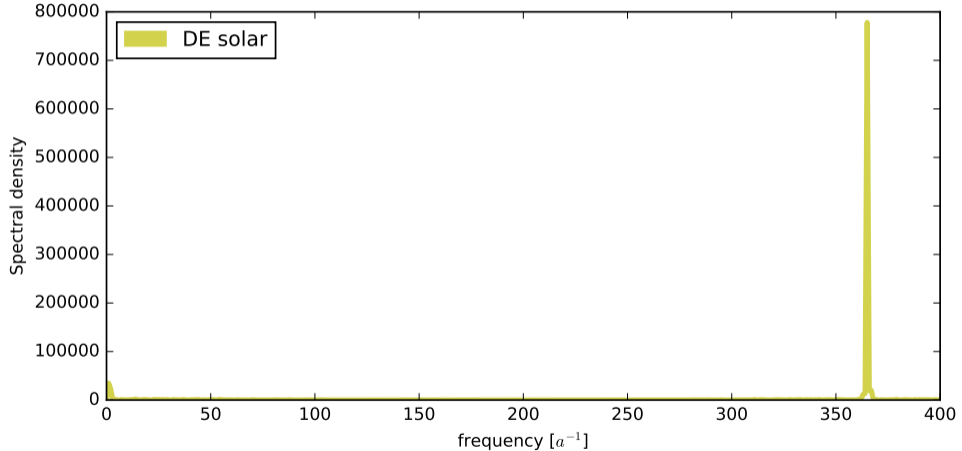
If we take a weekly average we see higher solar in the summer.



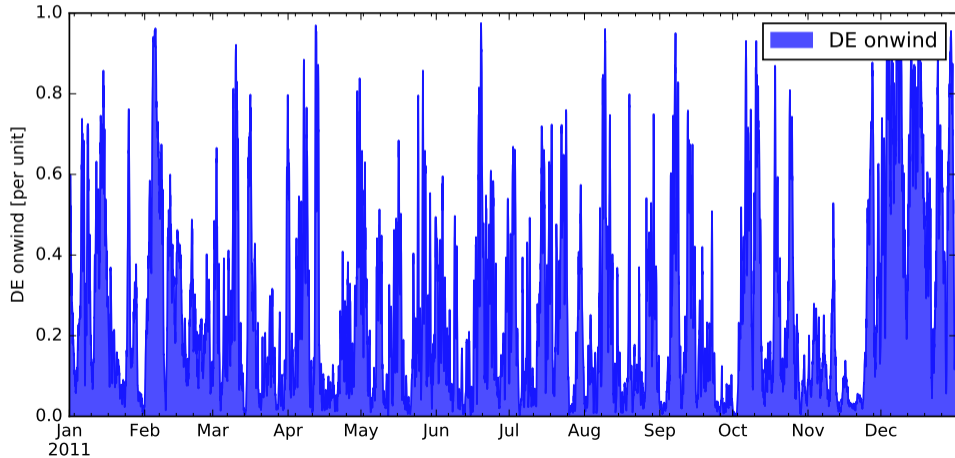




If we Fourier transform, the **seasonal** and **daily** patterns become visible.

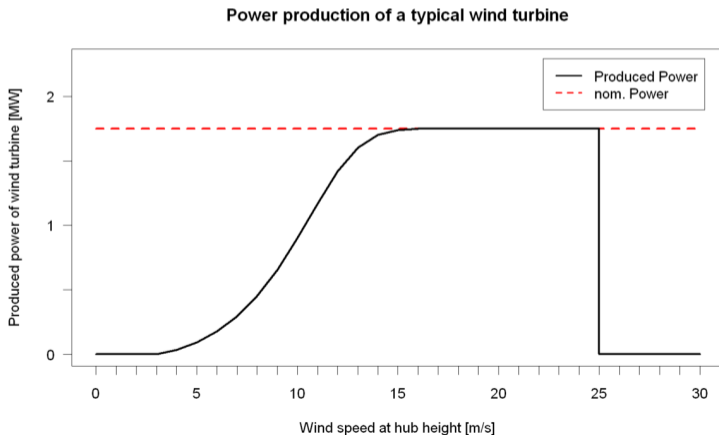


Wind is variable, like solar, but the variations are on different time scales. It drops close to zero and rarely reaches full output (when aggregated over all of Germany).



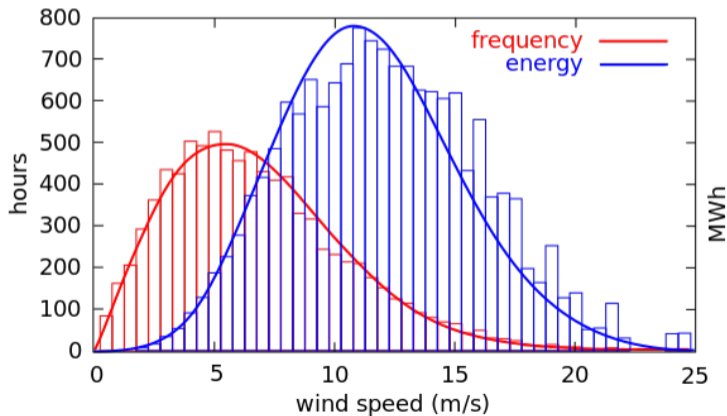
How do we derive wind time series?

We take times series weather data for the wind speeds at hub height (e.g. 60-100m) at each location in ms^{-1} . In theory the power in the wind goes like v^3 , but in practice high wind speeds are rare and it is not economic to build the generator so large.

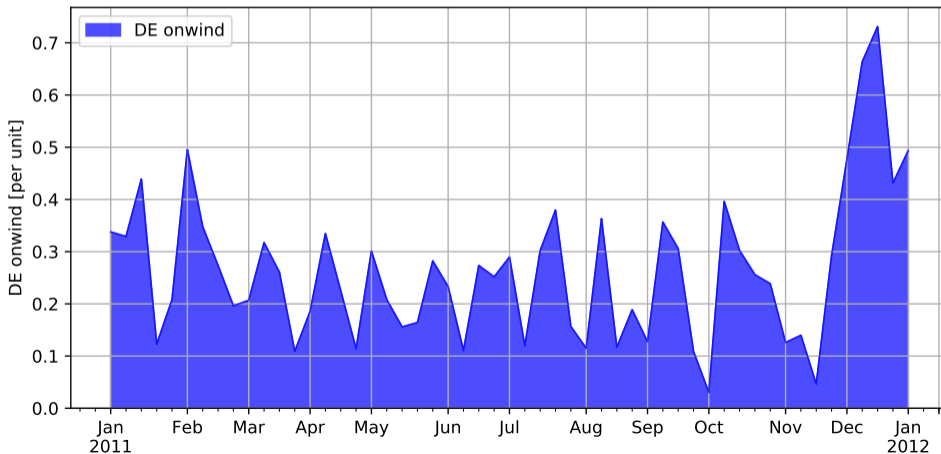


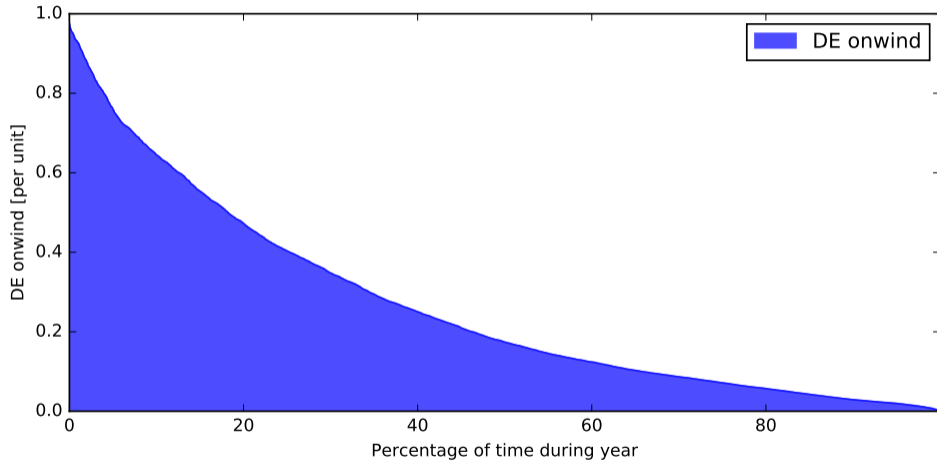
How do we derive wind time series?

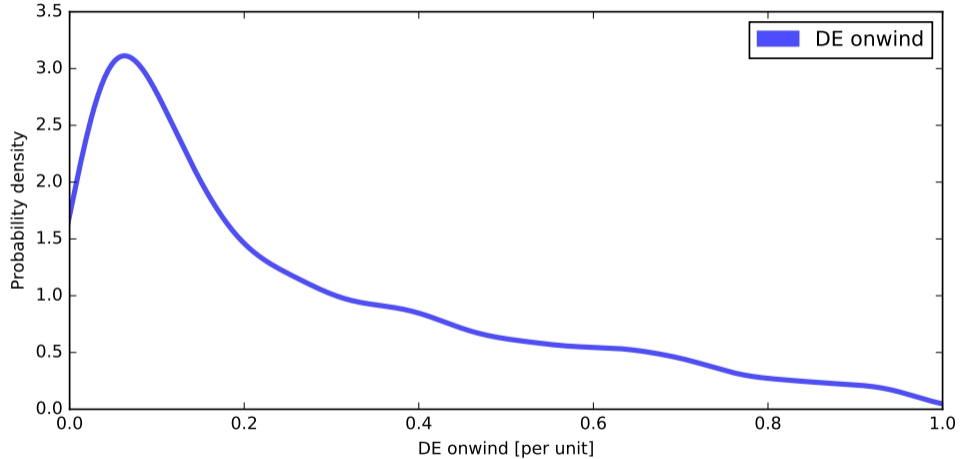
Wind speeds are typically distributed according to a Weibull probability distribution. Although the wind speeds are clustered at the lower end, most of the energy is generated between 5 and 15 ms^{-1} .



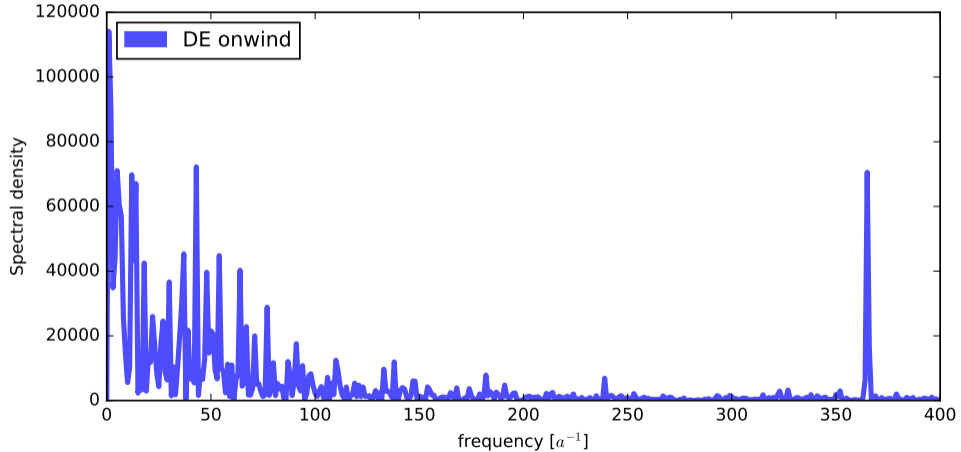
If we take a weekly average we see higher wind in the winter and some periodic patterns over 2-3 weeks (**weekly or synoptic scale**).



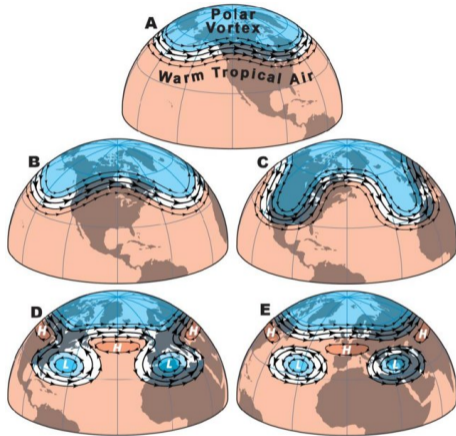




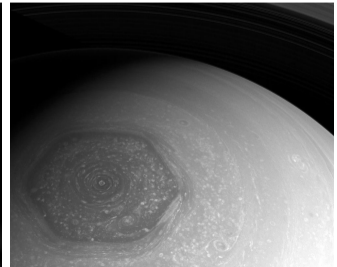
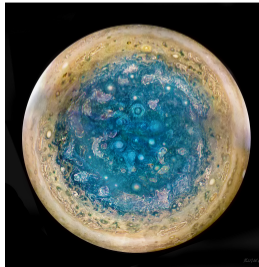
If we Fourier transform, the **seasonal**, **synoptic** (2-3 weeks) and **daily** patterns become visible.



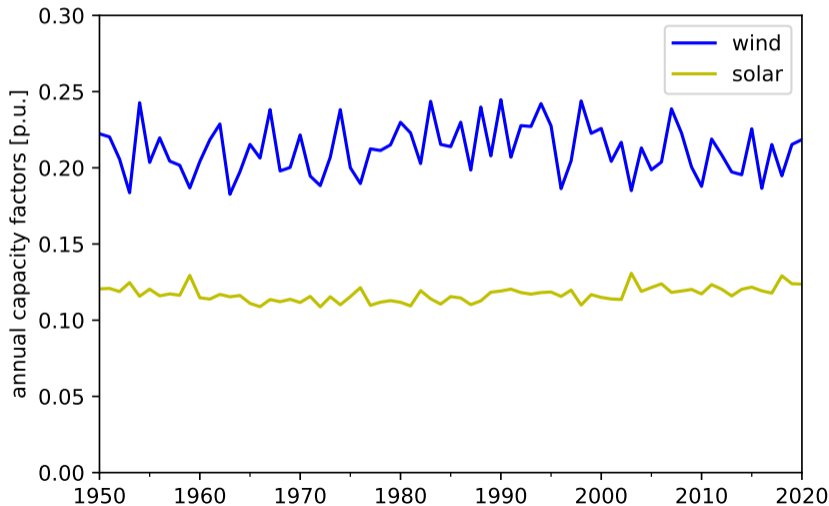
What causes weekly variations?



- Large weather systems follow atmospheric **Rossby waves**. Rossby waves are giant meanders in high-altitude winds that have a major influence on weather.
- Also found on other planets like Jupiter and Saturn (general feature of rotating fluids).



Particularly wind shows strong **inter-annual variability** (i.e. between different years).



Balancing a Single Country

Suppose we now try and cover the electrical demand with the generation from wind and solar.

How much wind and solar do we need? We have three time series:

- $\{d_t\}$, $d_t \in \mathbb{R}$ the load (varying between 35 GW and 80 GW)
- $\{w_t\}$, $w_t \in [0, 1]$ the wind availability (how much a 1 MW wind turbine produces)
- $\{s_t\}$, $s_t \in [0, 1]$ the solar availability (how much a 1 MW solar turbine produces)

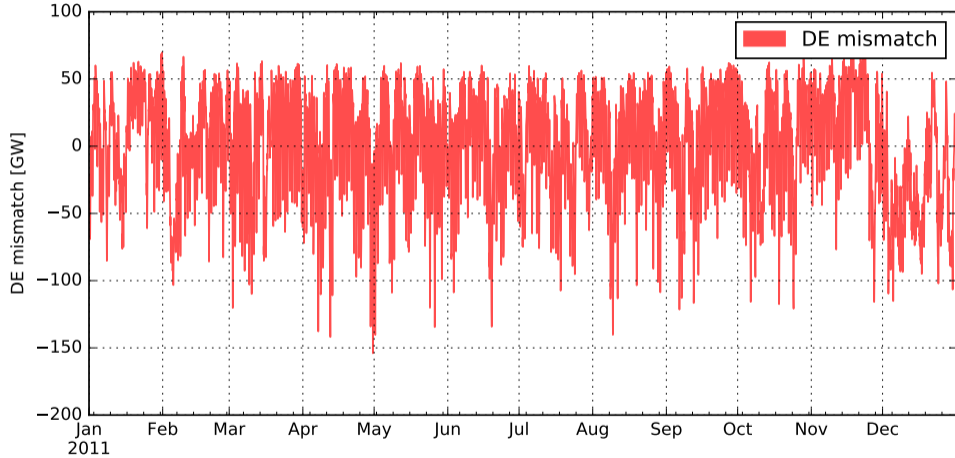
We try W MW of wind and S MW of solar. Now the effective **residual load** or **mismatch** is

$$m_t = d_t - Ww_t - Ss_t$$

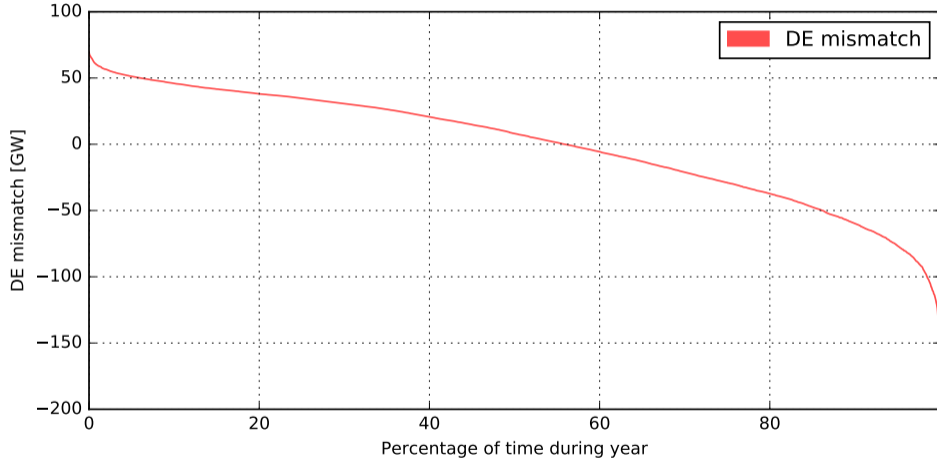
We choose W and S such that on **average** we cover all the load

$$\langle m_t \rangle = 0$$

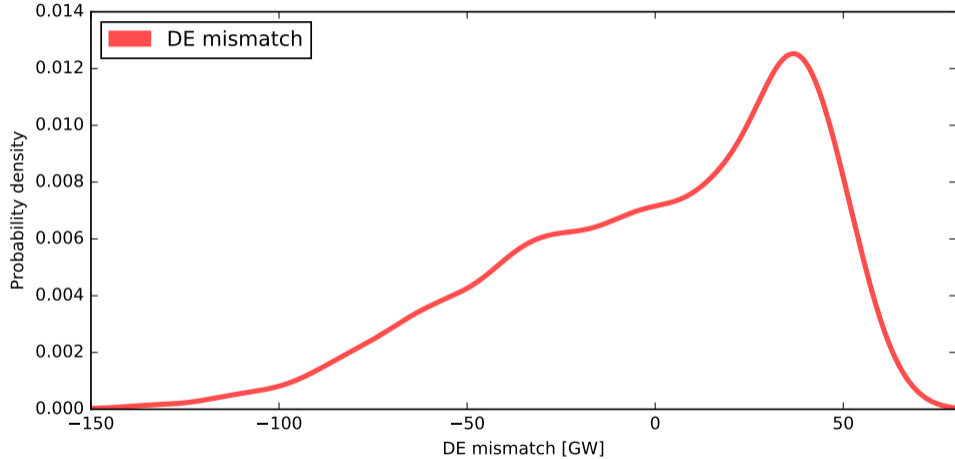
and so that the 70% of the energy comes from wind and 30% from solar ($W = 147$ GW and $S = 135$ GW).



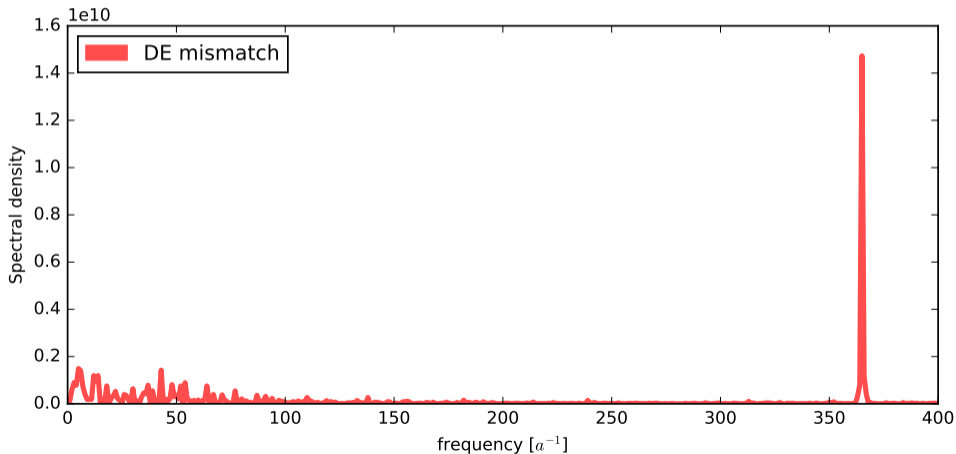
Mismatch duration curve



Mismatch density function



If we Fourier transform, the synoptic (from wind) and daily patterns (from demand and solar) become visible. Seasonal variations appear to cancel out.



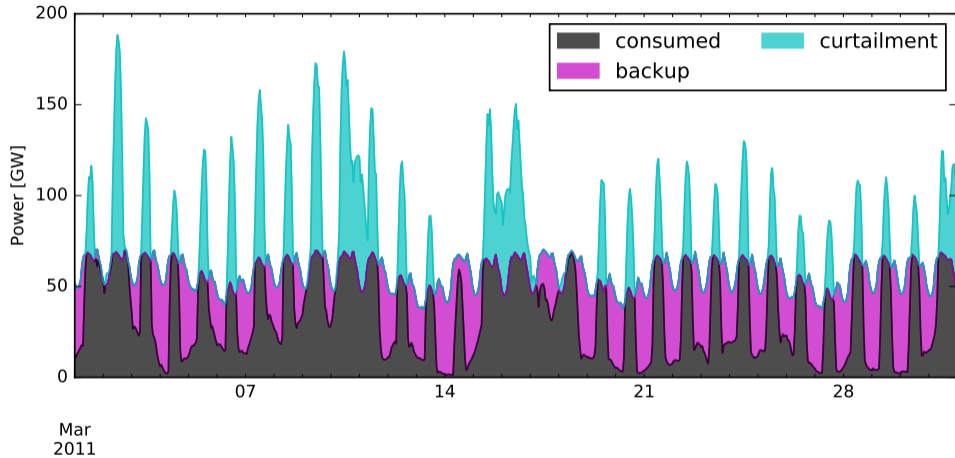
The problem is that

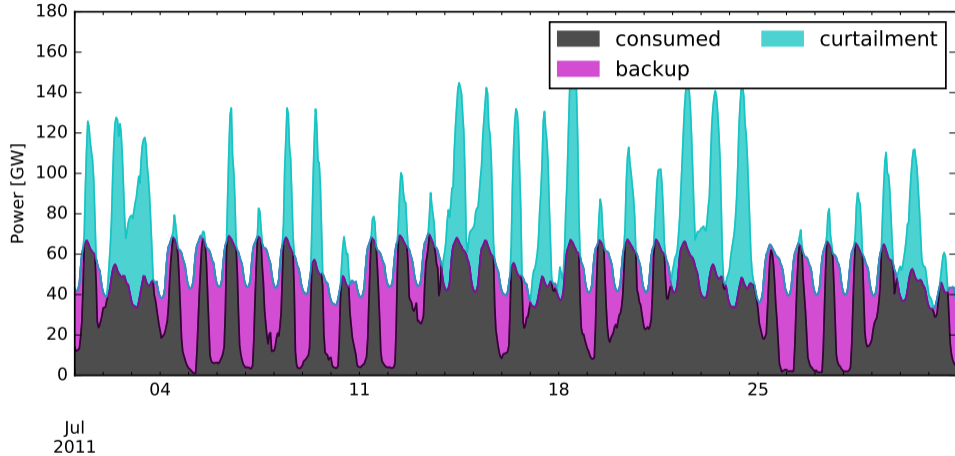
$$\langle m_t \rangle = 0$$

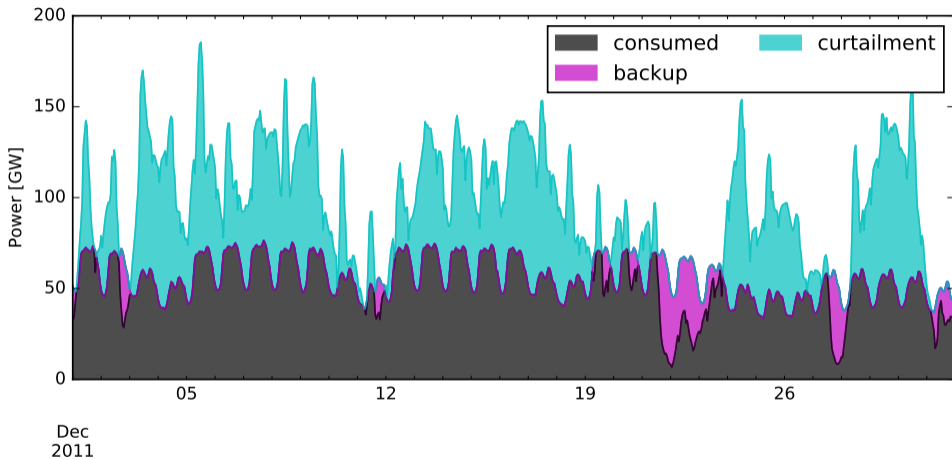
is not good enough! We need to meet the demand in every single hour.

This means:

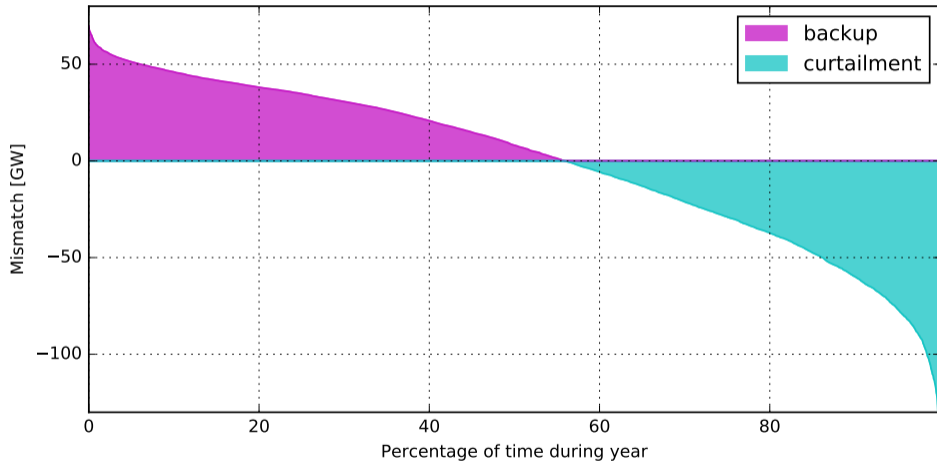
- If $m_t > 0$, i.e. we have unmet demand, then we need backup generation from **dispatchable** sources e.g. hydroelectricity reservoirs, fossil/biomass fuels.
- If $m_t < 0$, i.e. we have over-supply, then we have to shed / spill / **curtail** the renewable energy.







Mismatch duration curve



Backup energy costs money and may also cause CO₂ emissions.

Curtailing renewable energy is also a waste.

We'll look in the next lectures at **four other solutions**:

1. **Smoothing** stochastic variations of renewable feed-in **over continental areas**, e.g. the whole of Europe.
2. Using **electricity storage** to shift energy from times of surplus to times of deficit.
3. Shifting demand to different times, when renewables are abundant, i.e. **demand-side management** (DSM).
4. Consuming the electricity in **other sectors**, e.g. transport or heating, where there are further possibilities for DSM (battery electric vehicles, heat pumps) and cheap storage possibilities (e.g. thermal storage or power-to-gas as hydrogen or methane).