Energy Systems, Summer Semester 2025
Lecture 11: Cost Recovery & Renewables

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Cost Recovery in Long-Term Equilibrium

Cost recovery in the long-term equilibrium

We will demonstrate that all players in the power network (generators, storage and network operators) **recover their costs** in theory with perfect markets in long-term equilibrium and linear (actually convex) costs.

If they didn't cover their costs, they would leave the market.

If they made a profit, others would join the market and competition would reduce the profit.

This is a direct consequence of the investment equations we considered in Lecture 11.

We will discuss at the end why this **does not work in real life**, i.e. the consequences of imperfect markets, frictions and non-convexities.

Single node with optimised capacities and dispatch

Suppose we have generators labelled by s at a single node with **marginal costs** o_s arising from each unit of production $g_{s,t}$ and **capital costs** c_s that arise from fixed costs regardless of the rate of production (such as the investment in building capacity G_s). For a variety of demand values d_t in representative situation t we optimise the total annual system costs

$$\min_{\{g_{s,t}\},\{G_s\}} \left[\sum_s c_s G_s + \sum_{s,t} o_s g_{s,t} \right]$$

such that (NB: we now include availability hourly capacity factor $G_{s,t} \in [0,1]$ for wind/solar)

$$\sum_s g_{s,t} = d_t \qquad \leftrightarrow \qquad \lambda_t$$

$$-g_{s,t} \leq 0 \qquad \leftrightarrow \qquad \underline{\mu}_{s,t}$$

$$g_{s,t} - G_{s,t} G_s \leq 0 \qquad \leftrightarrow \qquad \bar{\mu}_{s,t}$$

We will now show using KKT that every generator exactly recovers their costs if the market price is set by λ_t^*, the **no/zero profit rule**.

Take the costs of generator s at the optimal point:

$$c_s G_s^* + \sum_t o_s g_{s,t}^*$$

Use stationarity for $g_{s,t}^*$

$$0 = \frac{\partial \mathcal{L}}{\partial g_{s,t}} = o_s - \lambda_t^* - \bar{\mu}_{s,t}^* + \underline{\mu}_{s,t}^*$$

to substitute for o_s in the costs:

$$c_s G_s^* + o_s \sum_t g_{s,t}^* = c_s G_s^* + \sum_t (\lambda_t^* + \bar{\mu}_{s,t}^* - \underline{\mu}_{s,t}^*) g_{s,t}^*$$

Single node with optimised capacities and dispatch

Next use complementarity

$$\bar{\mu}^*_{s,t}(g^*_{s,t} - G_{s,t}G^*_s) = 0$$
$$\underline{\mu}^*_{s,t}g^*_{s,t} = 0$$

to substitute for the terms $\mu^* g^*_{s,t}$

$$c_s G^*_s + o_s \sum_t g^*_{s,t} = c_s G^*_s + \sum_t (\lambda^*_t + \bar{\mu}^*_{s,t} - \underline{\mu}^*_{s,t}) g^*_{s,t}$$
$$= c_s G^*_s + \sum_t \lambda^*_t g^*_{s,t} + \sum_t \bar{\mu}^*_{s,t} G_{s,t} G^*_s$$

Finally use stationarity for the capacity G^*_s

$$0 = \frac{\partial \mathcal{L}}{\partial G_s} = c_s + \sum_t \bar{\mu}^*_{s,t} G_{s,t}$$

to get **full cost recovery** from the market price:

$$c_s G^*_s + o_s \sum_t g^*_{s,t} = \sum_t \lambda^*_t g^*_{s,t}$$

Network of nodes with optimised capacities and dispatch

Suppose now we have a network of nodes i connected by lines ℓ.

Our investment problem is now:

$$\min_{\{g_{i,s,t}\},\{G_{i,s}\},f_{\ell,t},F_\ell} \left[\sum_{i,s} c_s G_{i,s} + \sum_{i,s,t} o_s g_{i,s,t} + \sum_{\ell} c_\ell F_\ell \right]$$

such that

$$\begin{aligned}
\sum_s g_{i,s,t} - \sum_\ell K_{i\ell} f_{\ell,t} = d_{i,t} \qquad &\leftrightarrow \qquad \lambda_{i,t} \\
-g_{i,s,t} \leq 0 \qquad &\leftrightarrow \qquad \underline{\mu}_{i,s,t} \\
g_{i,s,t} - G_{i,s,t} G_{i,s} \leq 0 \qquad &\leftrightarrow \qquad \bar{\mu}_{i,s,t} \\
f_{\ell,t} - F_\ell \leq 0 \qquad &\leftrightarrow \qquad \bar{\mu}_{\ell,t} \\
-f_{\ell,t} - F_\ell \leq 0 \qquad &\leftrightarrow \qquad \underline{\mu}_{\ell,t}
\end{aligned}$$

The cost recovery of the generators follows through exactly as before.

What about the costs $c_\ell F_\ell^*$ of each transmission line?

Use stationarity for the capacity F_ℓ^*:

$$0 = \frac{\partial \mathcal{L}}{\partial F_\ell} = c_\ell + \sum_t \bar{\mu}_{\ell,t}^* + \sum_t \underline{\mu}_{\ell,t}^*$$

to get

$$c_\ell F_\ell^* = -F_\ell^* \sum_t \left[\underline{\mu}_{\ell,t}^* + \bar{\mu}_{\ell,t}^* \right]$$

'At the optimal point, fixed costs equal the sum of marginal benefits of expanding the line at each time.'

Next use complementarity for the flows $\bar{\mu}_{\ell,t}^*(f_{\ell,t}^* - F_\ell^*) = 0$ and $\underline{\mu}_{\ell,t}^*(-f_{\ell,t}^* - F_\ell^*) = 0$ to get

$$c_\ell F_\ell^* = -\sum_t \left[\bar{\mu}_{\ell,t}^* - \underline{\mu}_{\ell,t}^* \right] f_{\ell,t}^*$$

Network of nodes with optimised capacities and dispatch

Finally use stationarity for each $f^*_{\ell,t}$:

$$0 = \frac{\partial \mathcal{L}}{\partial f_{\ell,t}} = \sum_i \lambda^*_{i,t} K_{i\ell} - \bar{\mu}^*_{\ell,t} + \underline{\mu}^*_{\ell,t}$$

to substitute for the μ^*:

$$\begin{aligned} c_\ell F^*_\ell &= -\sum_t \left[\bar{\mu}^*_{\ell,t} - \underline{\mu}^*_{\ell,t}\right] f^*_{\ell,t} \\ &= -\sum_t \sum_i \lambda^*_{i,t} K_{i\ell} f^*_{\ell,t} \end{aligned}$$

$-\sum_i \lambda^*_{i,t} K_{i\ell} f^*_{\ell,t}$ is nothing other than the **congestion rent** on line ℓ at time t, i.e. the flow $f^*_{\ell,t}$ multiplied by the price difference across the line $\sum_i \lambda^*_{i,t} K_{i\ell}$.

At the long-term equilibrium, the network operator covers the costs of the line exactly with the congestion rent. The optimum requires congestion at least some of the time!

Storage cost recovery

The proof for storage is a bit grizzly, but you can find it in **this paper**.

The result is for storage unit r at node i:

$$c_{r,\text{discharge}} G^*_{i,r,\text{discharge}} + c_{r,\text{charge}} G^*_{i,r,\text{charge}} + c_{r,\text{energy}} E^*_{i,r}$$
$$= \sum_t \lambda^*_t g^*_{i,r,t,\text{discharge}} - \sum_t \lambda^*_t g^*_{i,r,t,\text{charge}}$$

All the costs, including the costs of the electricity to charge the storage, are recovered when the storage discharges, thereby selling its electricity to the market.

At the equilibrium, the profits from arbitrage in the market ('buy low, sell high') exactly cover the investment costs.

From KKT we can deduce the optimal levels at which storage should bid into the market as demand or offer as supply (more later maybe).

Adding a CO2 constraint for a single node

If we add a constraint on the total CO_2 emissions

$$\sum_{s,t} \frac{\varepsilon_s}{\eta_s} g_{s,t} \leq \text{CAP} \leftrightarrow \mu_{CO2}$$

where ε_s are the specific CO_2 emissions of technology s per fuel thermal energy and η_s is the efficiency of the generator (i.e. the ratio between thermal energy and electrical energy). CAP could correspond to e.g. political targets for CO_2 reduction.

All that changes is stationarity for the generator

$$0 = \frac{\partial \mathcal{L}}{\partial g_{s,t}} = o_s - \lambda_t^* + \underline{\mu}_{s,t}^* - \bar{\mu}_{s,t}^* - \mu_{CO2}^* \frac{\varepsilon_s}{\eta_s}$$

and now for each generator cost recovery becomes

$$c_s G_s^* + o_s \sum_t g_{s,t}^* = \sum_t \lambda_t^* g_{s,t} + \mu_{CO2}^* \sum_t \frac{\varepsilon_s}{\eta_s} g_{s,t}^*$$

This shows nicely the duality for exchanging the CO2 constraint for a CO2 price $o_s \to o_s - \mu_{CO2}^* \frac{\varepsilon_s}{\eta_s}$ (remember $\mu_{CO2}^* \leq 0$ for minimisation problems).

Introduction to Lagrangian Relaxation

This switching between costs and constraints is a special case of **Lagrangian relaxation**.

Consider the optimisation problem:

$$\max_x f(x)$$

$[x = (x_1, \ldots x_k)]$ subject to some **constraints** within $\mathbb{R}^k$:

$$\begin{array}{cccc} g_i(x) = c_i & \leftrightarrow & \lambda_i & i = 1, \ldots n \\ h_0(x) \leq d_0 & \leftrightarrow & \mu_0 & \\ h_j(x) \leq d_j & \leftrightarrow & \mu_j & j = 1, \ldots m \end{array}$$

Now consider the related problem where $\tilde{\mu}_0$ is fixed to a constant:

$$\max_x f(x) - \tilde{\mu}_0(h_0(x) - d_0)$$

$[x = (x_1, \ldots x_k)]$ subject to some **constraints** within $\mathbb{R}^k$:

$$\begin{array}{cccc} g_i(x) = c_i & \leftrightarrow & \lambda_i & i = 1, \ldots n \\ h_j(x) \leq d_j & \leftrightarrow & \mu_j & j = 1, \ldots m \end{array}$$

We have **relaxed** the problem by removing one of the constraints.

You can show that the new problem has the same solution as the old (x^*, λ^*, μ^*) if we fix the constant $\tilde{\mu}_0 = \mu_0^*$ by comparing the KKT stationarity constraints of the two problems.

We have lifted the constraint into the objective function, where it penalises solutions with $h_0(x) > d_0$.

In general, if we don't know μ_0^* beforehand, we can iteratively solve to find it.

Often it can be easier to solve the relaxed problem.

Fundamental Welfare Theorem is Lagrangian Relaxation

Consider the maximisation of total welfare:

$$\max_{\{d_b\},\{g_s\}} f(\{d_b\},\{g_s\}) = \left[\sum_b U_b(d_b) - \sum_s C_s(g_s)\right]$$

subject to the balance constraint:

$$g(\{d_b\},\{g_s\}) = \sum_b d_b - \sum_s g_s = 0 \qquad \leftrightarrow \qquad \lambda$$

Now let's relax the constraint:

$$\max_{\{d_b\},\{g_s\}} f(\{d_b\},\{g_s\}) = \left[\sum_b U_b(d_b) - \sum_s C_s(g_s) - \tilde{\lambda}(\sum_b d_b - \sum_s g_s)\right]$$

This problem is **separable** and **decomposes** into separate problems for each d_b:

$$\max_{d_b} \left[U_b(d_b) - \tilde{\lambda} d_b\right]$$

and for each g_s:

$$\max_{g_s} \left[\tilde{\lambda} g_s - C_s(g_s)\right]$$

Grit in the machine for generation 1/2

Several factors make this theoretical picture quite different in reality:

- Generation investment is **lumpy** i.e. you can often only build power stations in e.g. 500 MW blocks, not at any size
- Some older generators have **sunk costs**, i.e. costs which have been incurred once and investments that cannot be reversed $\Rightarrow$ they have no incentive to withdraw from the market if they are no longer cost-optimal in the long-term
- **Returns on scale** in building plant are not taken into account (specific capital costs [€/kW] going down would be a non-convexity; we did everything linear)
- Site-specific concerns ignored (e.g. lignite might need to be near a mine and have limited capacity)
- Variability of production for wind/solar ignored
- There is considerable uncertainty given load/weather conditions during a year, which makes investment risky; economic downturns reduce electricity demand

Grit in the machine for generation 2/2

Several factors make this theoretical picture quite different in reality:

- Fuel cost fluctuations, building delays which cost money
- Risks from third-parties: Changing costs of other generators, political risks (CO_2 taxes, Atomausstieg, subsidies for renewables, price caps)
- Political or administrative constraints on wholesale energy prices may prevent prices from rising high enough for long enough to justify generation investment ("Missing Money Problem")
- Lead-in time for planning and building, behaviour of others, boom-and-bust investment cycles resulting from periods of under- and over-investment in capacity
- Exercise of **market power** - single companies can dominate the market and alter the price by changing their supply bids - they are no longer price takers

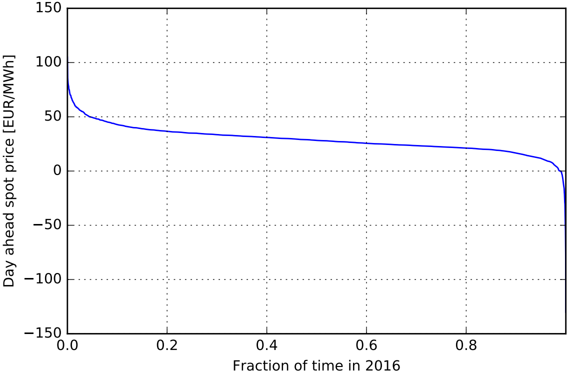
Episodes of High Prices are an Essential Part of an Energy-Only Market

In an energy-only market (in which generators are only compensated for the energy they produce), the wholesale spot price must at times be higher than the variable cost of the highest-variable-cost generating unit in the market. Episodes of high prices and/ or price spikes are not in themselves evidence of market power or evidence of market failure.

However, there may be political or administrative restrictions on prices going to very high levels (i.e. consumer protection, concerns about market abuse).

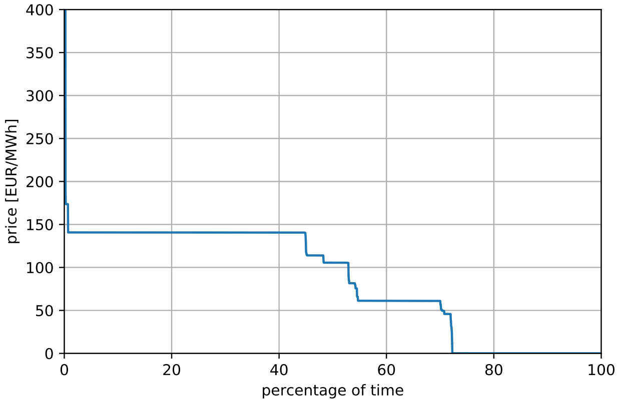
Today's market does not (usually) have enough times of high prices

This makes it hard for e.g. gas generators to make back their costs. Day ahead spot market prices in 2016 in Germany-Austria bidding zone:



Gas generators can bid into other markets, such as the intra-day or reserve power markets, or provide redispatch services.

In our simulations for high renewable penetrations (taken from **this paper**), the theory does however work:

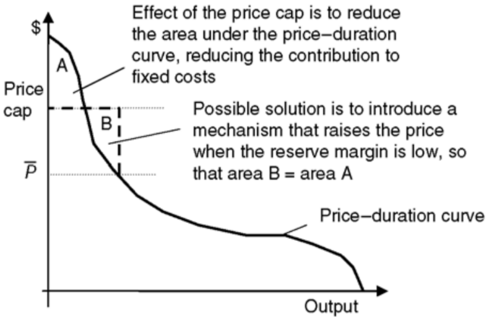


Prices are zero around a quarter of the time, but spike above 10,000 €/MWh in some hours.

Price cap

Some markets implement a maximum market price cap (MPC), which may be below the Value of Lost Load (VoLL) (V for the inelastic case).

In the Eastern Australian National Electricity Market (NEM), a MPC of A$15,000/MWh (€ 9,300/MWh) for the 2020-2021 financial year is set, corresponding to the price automatically triggered when AEMO directs network service providers to interrupt customer supply in order to keep supply and demand in the system in balance.



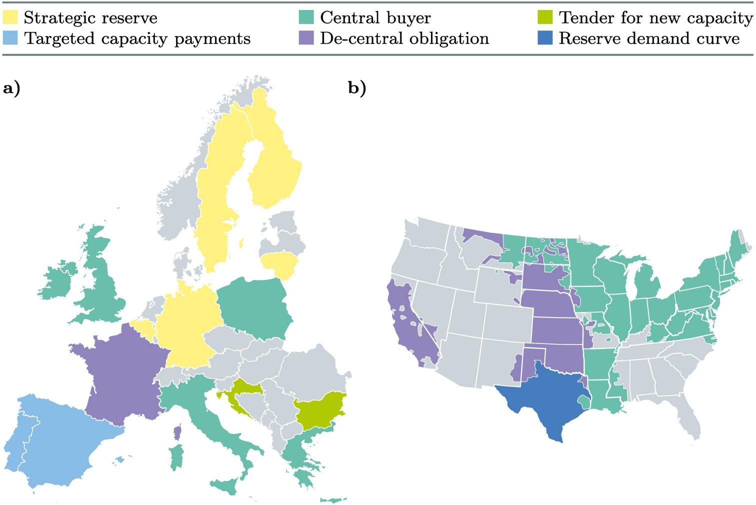
The Electric Reliability Council of Texas (ERCOT) has an energy only market with an MPC of $9000/MWh.

MPC can introduce distortions which make it difficult for some generators to recover costs.

Source: Biggar and Hesamzadeh, 2014

Capacity Remuneration Mechanisms vary widely

Capacity Remuneration Mechanisms (CRM) in 2019 in Europe and the US:



Source: Bublitz et al, 2019

Features of Transmission Investment 1/2

1. **Rationale for transmission**: Load and generation do not coincide in location at all times, so electricity must be transported for some of the time.
2. **Transmission is a natural monopoly**: Like railways or water provision, it is unlikely that a parallel electricity network would be built, given cost and limits on installing infrastructure due to space and public acceptance. Natural monopolies require **regulation**.
3. **Transmission is a capital-intensive business**: Transmitting electric power securely and efficiently over long distances requires large amounts of equipment (lines, transformers, etc.) which dominate costs compared to the operating costs of the grid. Making good investment decisions is thus the most important aspect of running a transmission company.

Features of Transmission Investment 2/2

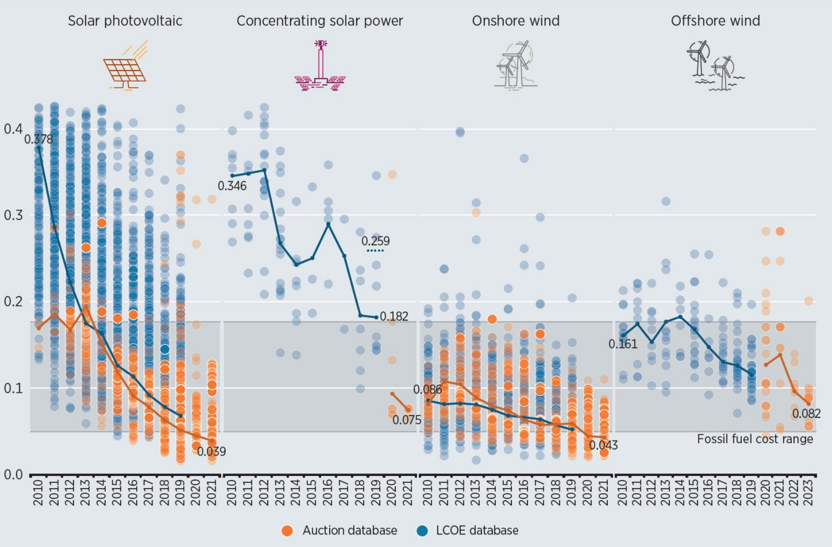
1. **Transmission assets have a long life**: Most transmission equipment is designed for an expected life ranging from 20 to 40 years or even longer (up to 60-80 years). A lot can change over this time, such as load behaviour and generation costs and composition.
2. **Transmission investments are irreversible**: Once a transmission line has been built, it cannot be redeployed in another location where it could be used more profitably.
3. **Transmission investments are lumpy**: Manufacturers sell transmission equipment in only a small number of standardized voltage and MVA ratings. It is therefore often not possible to build a transmission facility whose rating exactly matches the need.
4. **Economies of scale**: Transmission investment more proportional to length (costs of rights of way, terrain, towers, which dominate costs) than to power rating (which depends only on conductoring, which is cheap).

Integrating Renewables in Power Markets

Characteristics of Renewables

- **Variability**: Their production depends on weather (wind speeds for wind, insolation for solar and precipitation for hydroelectricity)
- **No Upwards Controllability**: Variable Renewable Energy (VRE) like wind and solar can only reduce their output; raising is hard
- **No Long-Term Forecastability**: Although short-term forecasting is improving steadily
- **Low Marginal Cost** (no fuel costs)
- **High Capital Cost**
- **No Direct Carbon Dioxide Emissions** (but some indirect ones from manufacturing)
- **Small unit size** (onshore wind turbine is 3-5 MW; coal/nuclear is 1000 MW)
- **Somewhat Decentralised Distribution** for some VRE (e.g. solar panels on household rooves); offshore is however very centralised
- **Provision of system services**: Increasing

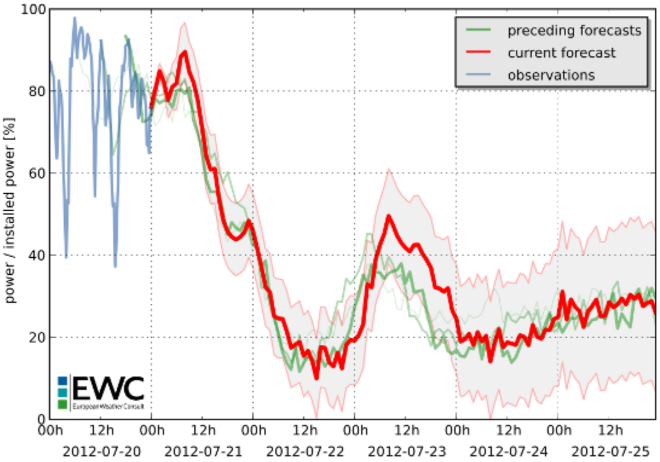
RE Levelised Cost in 2019 USD/kWh (already at/below fossil fuels)



For CSP, the dashed blue bar in 2019 shows the weighted average value including projects in Israel.

Source: **IRENA Renewable Generation Costs in 2019**

Just like the weather on which it depends, Variable RE (wind and solar) production can be forecast in advance. (Shaded area is the uncertainty.)



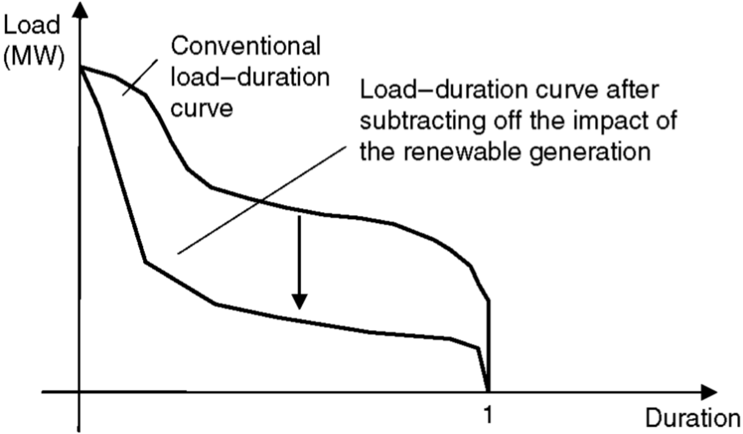
Like the weather, the forecast in the short-term (e.g. day ahead) is fairly reliable, particularly for wind, but for several days ahead it is less useful. In addition, it is subject to more uncertainty than the load. For example, fog and mist is very local, hard to predict, and has a big impact on solar power production.

This makes scheduling more challenging and has led to the introduction of more regular auctions in the intraday market.

Forecasting has also become a big business.

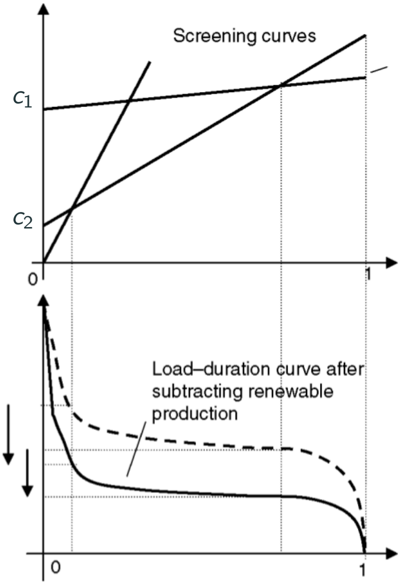
Effect on effective 'residual' load curve

Since RE often have priority feed-in (i.e. network operators are obliged to take their power), we often subtract the RE production from the load to get the **residual load**, plotted here as a demand-duration-curve.



Source: Biggar and Hesamzadeh, 2014

Residual load curve and screening curve



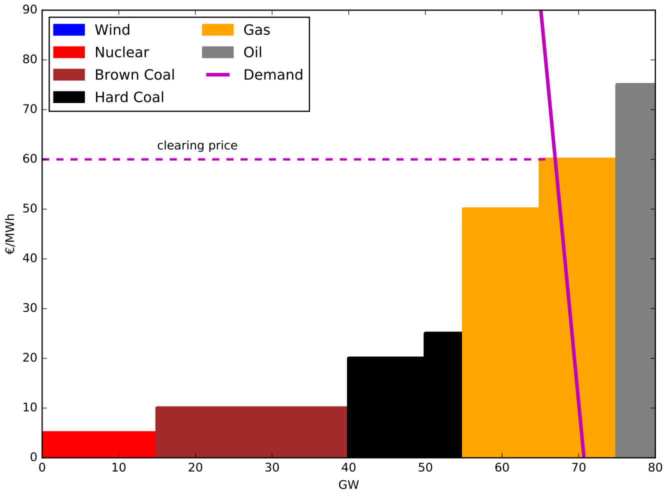
The residual load must be met by conventional generators.

The changed duration curve interacts differently with the screening curve, so that we may require less baseload generation and peaking plant and more load shedding, depending on the shape of the curve.

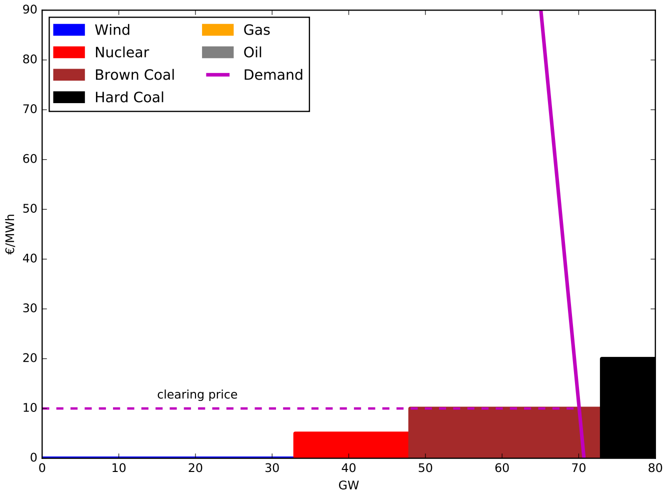
In some markets, there is increased demand for medium-peaking plant.

Source: Biggar and Hesamzadeh, 2014

Effect of varying renewables: fixed demand, no wind

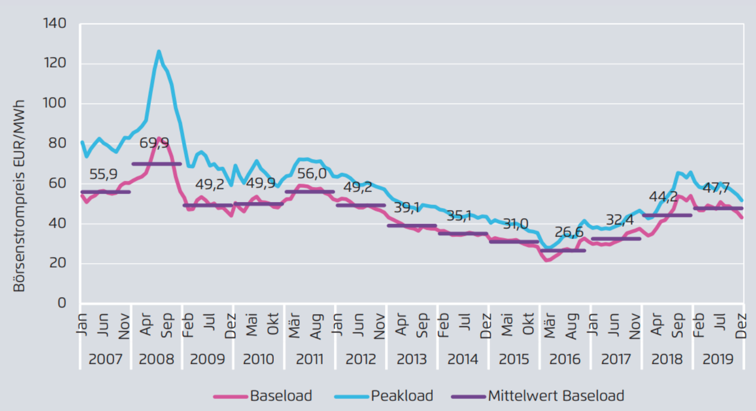


Effect of varying renewables: fixed demand, 35 GW wind



Spot market price development

As a result of so much zero-marginal-cost renewable feed-in, spot market prices steadily decreased until 2016. This is called the **Merit Order Effect**. Since then prices have been rising due to rising gas and CO_2 prices.



Source: Agora Energiewende Jahresauswertung 2019

To summarise:

- Renewables have zero marginal cost
- As a result they enter at the bottom of the merit order, reducing the price at which the market clears
- This pushes non-CHP gas and hard coal out of the market
- This is unfortunate, because among the fossil fuels, gas is the most flexible and produces lower CO_2 per MWh_{el} than e.g. lignite
- It also reduces the profits that nuclear and lignite make
- Will there be enough backup power plants for times with no wind/solar?

This has led to lots of political tension, but has been counteracted in recent years by the rising CO_2 price.

Market value

VRE have the property that they cannibalise their own market, by pushing down prices when lots of other VRE are producing.

We define the **market value** of a technology by the average market price it receives when it produces, i.e.

$$MV_s = \frac{\sum_t \lambda_t^* g_{s,t}}{\sum_t g_{s,t}}$$

We can compare this to the average market price, defined either as the simple average $\frac{1}{T}\sum_t \lambda_t^*$ or the demand-weighted average $\frac{\sum_t \lambda_t^* d_t}{\sum_t d_t}$.

Historic market values in Germany

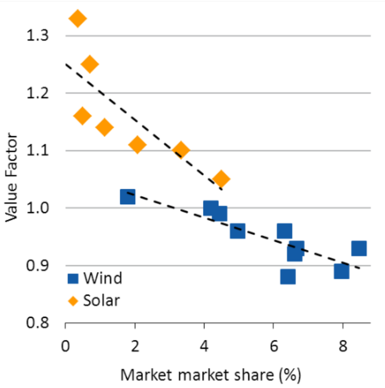


Figure 6. Historical wind and solar value factors in Germany (as reported numerically in Table 3).

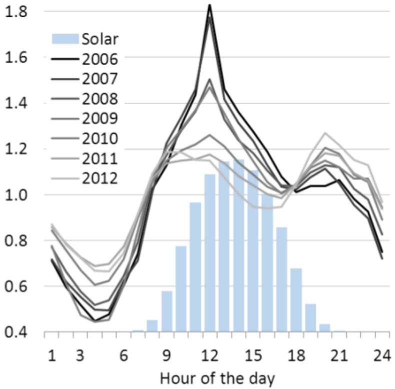
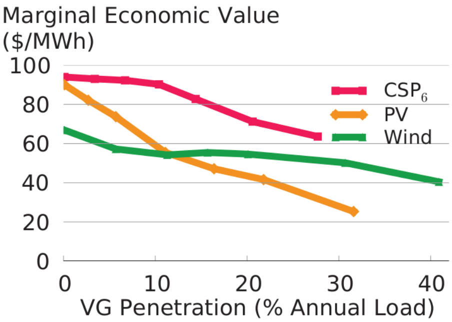


Figure 7. The daily price structure in Germany during summers from 2006 – 2012. The bars display the distribution of solar generation over the day.

Source: Lion Hirth, 2013

Market value at higher shares

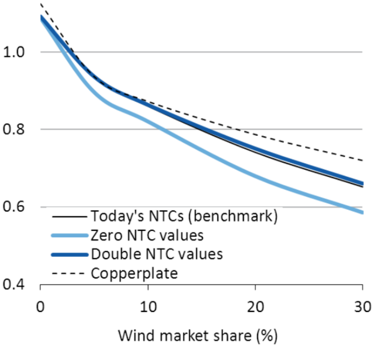
At low shares of VRE the market value may be higher than the average market price (because for example, PV produces a midday when prices are higher than average), but as VRE share increases the market value goes down.

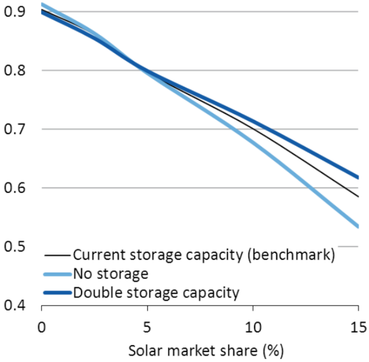


The effect is particularly severe for PV, since the production is highly correlated; for wind smoothing prevents a steeper drop off. The bigger the catchment area, the longer wind preserves its market value.

Source: Mills & Wiser, 2014

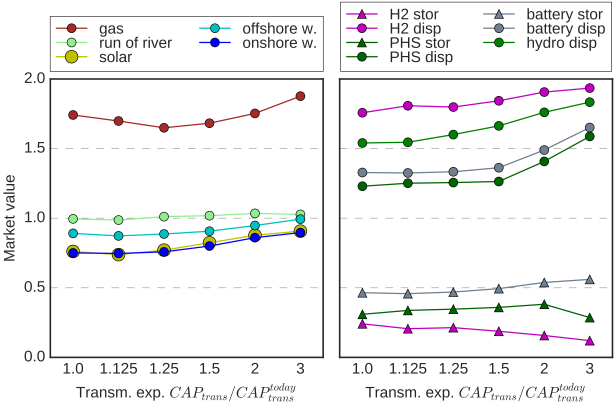
To halt the drop in market value (and hence revenue for wind and solar) we can use networks to do price arbitrage in space, storage to do arbitrage in time, or introduce CO2 prices that push up the prices in times when fossil fuel plants are running.





Source: Lion Hirth, 2013

Market value from our 95% renewable simulations



- Storage charges at low market prices and dispatches at high prices.
- Dispatchable power sources take advantage of high prices.
- Variable renewables get lower prices, but saved by storage, networks and high CO2 price.

Relation of LCOE to market value

From the first section we had for a perfect market in long-term equilibrium that all costs are recovered from market revenue:

$$c_s G_s^* + o_s \sum_t g_{s,t}^* = \sum_t \lambda_t^* g_{s,t}^*$$

If we divide both sides by the total yearly generation $\sum_t g_{s,t}^*$ then we get:

$$\frac{c_s G_s^* + o_s \sum_t g_{s,t}^*}{\sum_t g_{s,t}^*} = \frac{\sum_t \lambda_t^* g_{s,t}^*}{\sum_t g_{s,t}^*}$$

This is none other than the identity between the LCOE and market value:

$$LCOE = MV$$

This *only* holds in a perfect equilibrium. I.e. the equilibrium is found by increasing the penetration until the market value equals the LCOE.

In reality the market is far from equilibrium: subsidies support technologies (with a longer-term view of pushing them down the learning curve), there are sunk costs for existing plants, excess capacity supported outside the energy-only market, etc.

For more details on market value, the zero profit rule and how market value is affected by CO_2 prices and VRE subsidies, see the paper

Decreasing market value of variable renewables can be avoided by policy action (2020)

This paper examines how to maintain market value even at high shares of wind and solar.

Value of hydrogen storage

Value of hydrogen

We can now use the machinery of shadow prices to explore the **value of hydrogen** and how it relates to the value of electricity in a system with hydrogen storage.

Suppose we have electrolysis with efficiency η_e and a fuel cell or hydrogen turbine with efficiency η_f. They have respective market values in the electricity market of MV_e and MV_f.

The value of hydrogen is given by the KKT multiplier of the storage constraint

$$e_t = e_{t-1} + \eta_e g_{e,t} - \eta_f^{-1} g_{f,t} \qquad \leftrightarrow \qquad \tilde{\lambda}_t$$

where e_t is the state of charge (amount of hydrogen at time t), $g_{e,t}$ is the power consumption of the electrolyser and $g_{f,t}$ is the electricity dispatch of the fuel cell or hydrogen turbine.

$\tilde{\lambda}_t$ is the **price/value of hydrogen** since it tells us the change in objective function if we increase hydrogen use in this hour.

Value of hydrogen turbine

The KKT stationarity for the discharge variable of the fuel cell or hydrogen turbine $g_{f,t}$ is

$$0 = \frac{\partial \mathcal{L}}{\partial g_{f,t}} = \eta_f^{-1}\tilde{\lambda}_t^* - \lambda_t^* + \underline{\mu}_{f,t}^* - \bar{\mu}_{f,t}^* \quad \forall t$$

Note that this has exactly the same structure as a conventional generator with marginal cost $o_f = \eta_f^{-1}\tilde{\lambda}_t^*$ based on a fuel cost $\tilde{\lambda}_t^*$. This sets how the storage bids into the electricity market.

Now multiply this equation by $g_{f,t}^*$, sum over t, then divide by $\sum_t g_{f,t}^*$.

$$0 = \eta_f^{-1}\frac{\sum_t \tilde{\lambda}_t^* g_{f,t}^*}{\sum_t g_{f,t}^*} - \frac{\sum_t \lambda_t^* g_{f,t}^*}{\sum_t g_{f,t}^*} + \frac{\sum_t \underline{\mu}_{f,t}^* g_{f,t}^*}{\sum_t g_{f,t}^*} - \frac{\sum_t \bar{\mu}_{f,t}^* g_{f,t}^*}{\sum_t g_{f,t}^*}$$

The 1st term is the turbine-demand-averaged hydrogen price $\langle \tilde{\lambda}_t^* \rangle_f$; the 2nd term is the market value of the turbine; the 3rd term vanishes by complementarity and by complementarity plus stationarity for G_f we have $\sum_t \bar{\mu}_{f,t}^* g_{f,t}^* = \sum_t \bar{\mu}_{f,t}^* G_f = -c_f G_f$. Rearranging

$$MV_f = \frac{c_f G_f}{\sum_t g_{f,t}} + \frac{\langle \tilde{\lambda}_t^* \rangle_f}{\eta_f} = LCOE_f$$

This is the LCOE of the hydrogen turbine (average capital cost plus the marginal cost).

Value of hydrogen electrolyser

Doing the same for the power consumption of the electrolyser $g_{e,t}$ we get from stationarity

$$0 = \frac{\partial \mathcal{L}}{\partial g_{e,t}} = -\eta_e \tilde{\lambda}_t^* + \lambda_t^* + \underline{\mu}_{e,t}^* - \bar{\mu}_{e,t}^* \quad \forall t$$

Note that this has exactly the same structure as a flexible demand bidding with a willingness to pay of $\eta_e \tilde{\lambda}_t^*$. The electrolyser is willing to pay up to $\eta_e \tilde{\lambda}_t^*$ for electricity because if it wants to produce 1 MWh of hydrogen, it needs $1/\eta_e$ MWh of electricity. If it pays $\eta_e \tilde{\lambda}_t^*$ €/MWh or less for $1/\eta_e$ MWh it will pay up to λ_t^* and still can make a profit in the hydrogen market.

If we do the same tricks by multiplying by $g_{e,t}^*$, summing over t and dividing by $\sum_t g_{e,t}^*$ we get

$$0 = -\eta_e \langle \tilde{\lambda}_t^* \rangle_e + MV_e + \frac{c_e G_e}{\sum_t g_{e,t}}$$

Rearranging and dividing by η_e we have

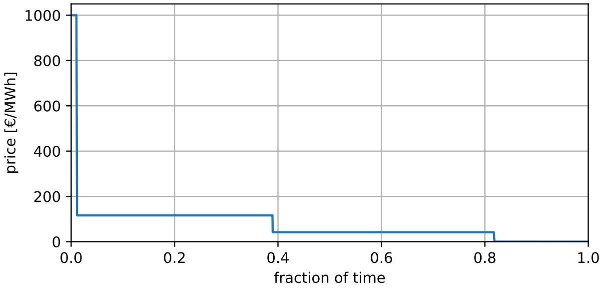
$$\langle \tilde{\lambda}_t^* \rangle_e = \frac{c_e G_e}{\eta_e \sum_t g_{e,t}} + \frac{MV_e}{\eta_e} = LCOH$$

The average value of hydrogen is the levelised cost of hydrogen, LCOH, at equilibrium, i.e. the averaged capital cost of the electrolyser plus the average cost of electricity used.

Consider the example in this **Jupyter notebook**. This runs over a year of weather for Germany with a flat demand met by wind, solar, hydrogen storage and load-shedding for 1000 €/MWh.

The hydrogen price is give by $\tilde{\lambda}_t^* = 67.3$ €/MWh. It is constant in time because we set the storage cost to zero $c_s = 0$ so that hydrogen can be moved between hours with no cost.

If we look at the electricity price duration curve for λ_t^*



Example calculation

What causes these price steps?

- The high price 1% of hours of 1000 €/MWh are set by the load-shedding.
- The next highest step for 38% of the time is set by the hydrogen turbine bidding as a supplier with marginal cost of $\eta_f^{-1}\tilde{\lambda}_t^* = (1/0.58) * 67.3$ €/MWh $= 116.0$ €/MWh.
- The next step for 43% of the time is set by the hydrogen electrolyser bidding as a demand with willingness to pay of $\eta_e\tilde{\lambda}_t^* = 0.62 * 67.3$ €/MWh $= 41.7$ €/MWh.
- For 18% of the time the price is set by wind and solar with zero marginal cost.

The hydrogen turbine has market value $MV_f = 162.2$ €/MWh and recovers its capital costs in the hours of load-shedding.

The electrolyser has market value $MV_e = 22.3$ €/MWh and recovers its capital costs by selling into the hydrogen market when the electricity price is zero.

Wind and solar have market values of 46.1 and 28.7 €/MWh respectively.

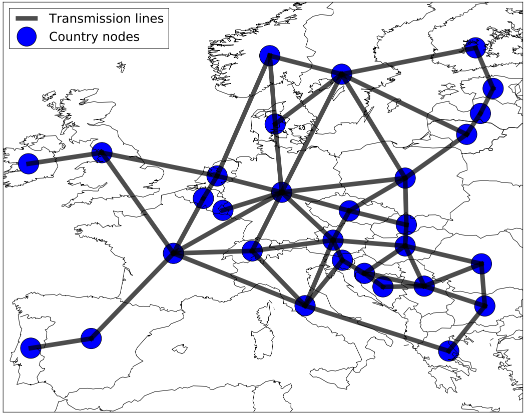
Hydrogen storage lessons

- First, this is only a **simulation of a zero-emission system** - we don't know exactly how the world will turn out (we won't have perfect foresight or equilibrium)
- Electricity prices are **intimately tied to hydrogen prices** (sector-coupling, just as prices are tied to fossil gas today)
- Unlike today, prices in many hours are **set by the demand side** (1% by load-shedding, 43% by electrolysers in our example; in a more complex world, prices could be set by flexible electric vehicle charging, stationary battery charging, heat pumps, flexible industrial loads)
- As a result of demand flexibility, prices are set by wind and solar to zero **only for a small fraction of hours** (here 18%)

Networks Versus Storage for Highly-Renewable European Electricity System

Warm-up: Determine optimal electricity system

- Meet all electricity demand.
- Reduce CO_2 by 95% compared to 1990.
- **Generation** (where potentials allow): onshore and offshore wind, solar, hydroelectricity, backup from natural gas.
- **Storage**: batteries for short term, electrolyse hydrogen gas for long term.
- **Grid expansion**: simulate everything from no grid expansion (like a **decentralised solution**) to optimal grid expansion (with significant **cross-border trade**).



Source: PyPSA-Eur, based on ENTSO-E map

Linear optimisation problem

Objective is the minimisation of **total annual system costs**, composed of **capital costs** c_* (investment costs) and **operating costs** o_* (fuel ,etc.):

$$\min f(F_\ell, f_{\ell,t}, G_{i,s}, g_{i,s,t}) = \sum_\ell c_l F_\ell + \sum_{i,s} c_{i,s} G_{i,s} + \sum_{i,s,t} w_t o_{i,s} g_{i,s,t}$$

We optimise for i nodes, representative times t and transmission lines l:

- the transmission capacity F_ℓ of all the lines ℓ
- the flows $f_{\ell,t}$ on each line ℓ at each time t
- the generation and storage capacities $G_{i,s}$ of all technologies (wind/solar/gas etc.) s at each node i
- the dispatch $g_{i,s,t}$ of each generator and storage unit at each point in time t

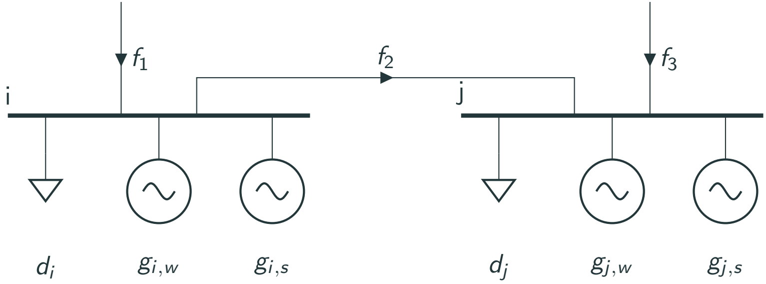
Representative time points are weighted w_t such that $\sum_t w_t = 365 * 24$ and the capital costs c_* are annualised, so that the objective function represents the annual system cost.

Constraints 1/6: Nodal energy balance

Demand $d_{i,t}$ at each node i and time t is always met by generation/storage units $g_{i,s,t}$ at the node or from transmission flows $f_{\ell,t}$ on lines attached at the node (Kirchhoff's Current Law):

$$\sum_s g_{i,s,t} - d_{i,t} = \sum_\ell K_{i\ell} f_{\ell,t} \qquad \leftrightarrow \qquad \lambda_{i,t}$$

Nodes are shown as thick busbars connected by transmission lines (thin lines):

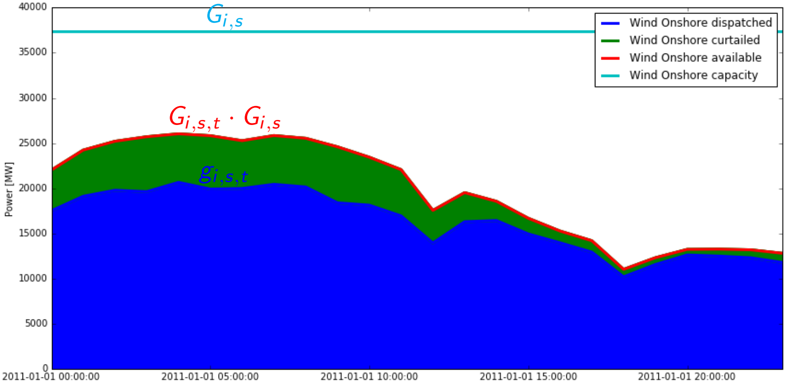


$$g_{i,w} + g_{i,s} - d_i = f_2 - f_1 \qquad g_{j,w} + g_{j,s} - d_j = -f_2 - f_3$$

Constraints 2/6: Generation availability

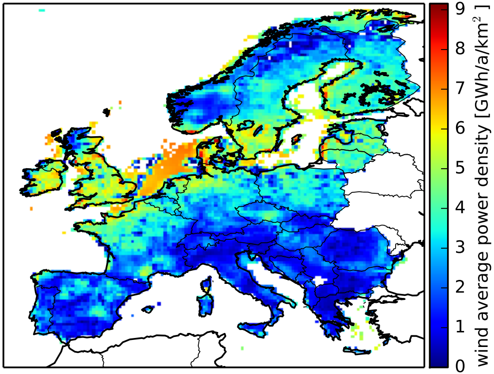
Generator/storage dispatch $g_{i,s,t}$ cannot exceed availability $G_{i,s,t} \cdot G_{i,s}$, made up of per unit availability $0 \leq G_{i,s,t} \leq 1$ multiplied by the capacity $G_{i,s}$. The capacity is bounded by the installable potential $\hat{G}_{i,s}$.

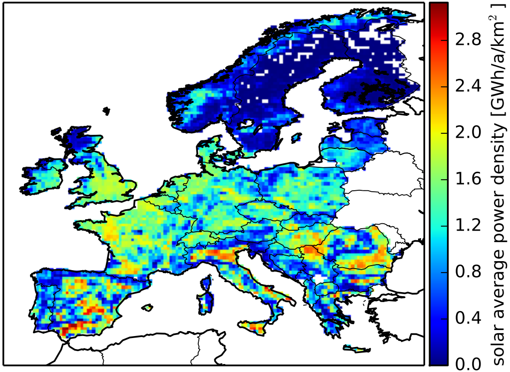
$$0 \leq g_{i,s,t} \leq G_{i,s,t} \cdot G_{i,s} \leq G_{i,s} \leq \hat{G}_{i,s}$$



Installation potentials limited by geography

Expansion potentials are limited by **land usage** and **conservation areas**; potential yearly energy yield at each site limited by **weather conditions**:





Constraints 3/6: Storage consistency

Storage units such as batteries or hydrogen storage can work in both storage and dispatch mode. This has to be consistent with the state of charge $e_{i,s,t}$:

$$e_{i,s,t} = \eta_0 e_{i,s,t-1} + \eta_1 g_{i,s,t,\text{store}} - \eta_2^{-1} g_{i,s,t,\text{dispatch}}$$

The state of charge is limited by the energy capacity $E_{i,s}$:

$$0 \leq e_{i,s,t} \leq E_{i,s} \quad \forall i, s, t$$

There are efficiency losses η; hydroelectric dams can also have a river inflow.

Constraints 4/6: Kirchoff's Laws for Physical Flow

The linearised **power flows** f_ℓ for each line $\ell \in \{1, \dots L\}$ in an AC network are determined by the **reactances** x_ℓ of the transmission lines and the **net power injection** at each node p_i for $i \in \{1, \dots N\}$.

We have to satisfy Kirchoff's Laws, which can be compactly expressed using the **incidence matrix** $K \in \mathbb{R}^{N \times L}$ (boundary operator in homology theory) of the graph and the **cycle basis** $C \in \mathbb{R}^{L \times (L-N+1)}$ (kernel of K)

- Kirchoff's Current Law: $p_i = \sum_\ell K_{i\ell} f_\ell$
- Kirchoff's Voltage Law: $\sum_\ell C_{\ell c} x_\ell f_\ell = 0$

Constraints 5/6: Transmission Line Thermal Limits

Transmission flows cannot exceed the thermal capacities of the transmission lines (otherwise they sag and hit buildings/trees):

$$|f_{\ell,t}| \leq F_\ell$$

Constraints 6/6: Global constraints on CO_2 and transmission volumes

CO_2 limits are respected, given emissions $\varepsilon_{i,s}$ for each fuel source s:

$$\sum_{i,s,t} g_{i,s,t} \frac{\varepsilon_{i,s}}{\eta_s} \leq \text{CAP}_{\text{CO}_2} \qquad \leftrightarrow \qquad \mu_{\text{CO}_2}$$

We enforce a reduction of CO_2 emissions by 95% compared to 1990 levels, in line with German and EU targets for 2050.

Transmission volume limits are respected, given length d_ℓ and capacity F_ℓ of each line:

$$\sum_{\ell} d_\ell F_\ell \leq \text{CAP}_{\text{trans}} \qquad \leftrightarrow \qquad \mu_{\text{trans}}$$

We successively change the transmission limit, to assess the costs of balancing power in time (i.e. storage) versus space (i.e. transmission networks).

Model Inputs and Outputs

Inputs	Description
$d_{i,t}$	Demand (inelastic)
$G_{i,s,t}$	Per unit availability for wind and solar
$\hat{G}_{i,s}$	Generator installable potentials
various	Existing hydro data
various	Grid topology
η_*	Storage efficiencies
$c_{i,s}$	Generator capital costs
$o_{i,s,t}$	Generator marginal costs
c_ℓ	Line costs

$\rightarrow$

Outputs	Description
$G_{i,s}$	Generator capacities
$g_{i,s,t}$	Generator dispatch
F_ℓ	Line capacities
$f_{\ell,t}$	Line flows
λ_*, μ_*	Lagrange/KKT multipliers of all constraints
f	Total system costs

Costs and assumptions for the electricity sector (projections for 2030)

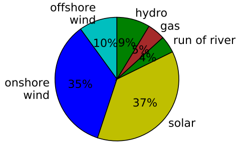
Quantity	Overnight Cost [€]	Unit	FOM [%/a]	Lifetime [a]
Wind onshore	1182	kW_{el}	3	20
Wind offshore	2506	kW_{el}	3	20
Solar PV	600	kW_{el}	4	20
Gas	400	kW_{el}	4	30
Battery storage	1275	kW_{el}	3	20
Hydrogen storage	2070	kW_{el}	1.7	20
Transmission line	400	MWkm	2	40

Interest rate of 7%, storage efficiency losses, only gas has CO_2 emissions, gas marginal costs.

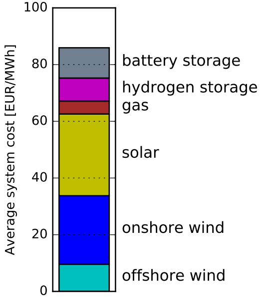
Batteries can store for 6 hours at maximal rating (efficiency 0.9×0.9), hydrogen storage for 168 hours (efficiency 0.75×0.58).

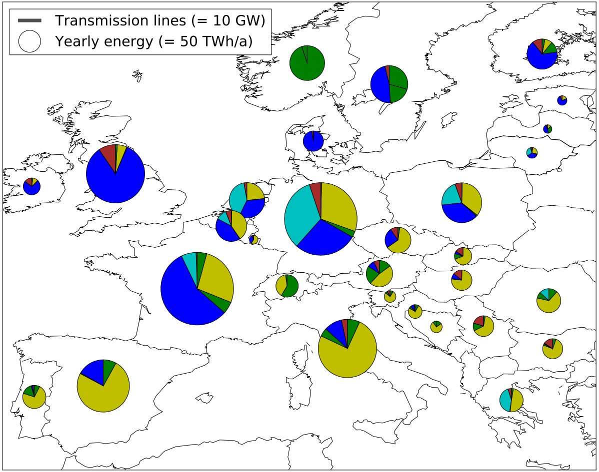
Costs: No interconnecting transmission allowed

Technology by energy:



Average cost **€86/MWh**:

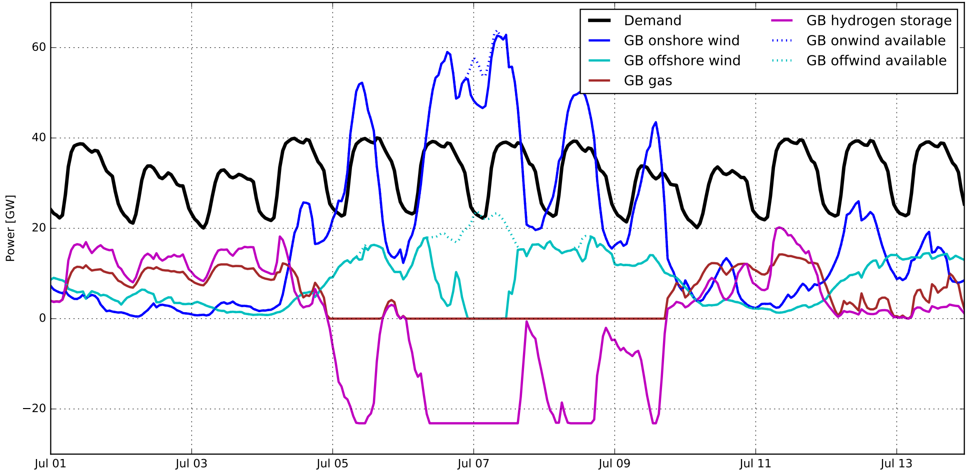




Countries must be self-sufficient at all times; lots of storage and some gas to deal with fluctuations of wind and solar.

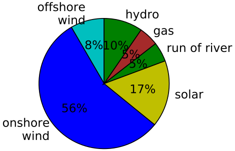
Dispatch with no interconnecting transmission

For Great Britain with no interconnecting transmission, excess wind is either stored as hydrogen or curtailed:

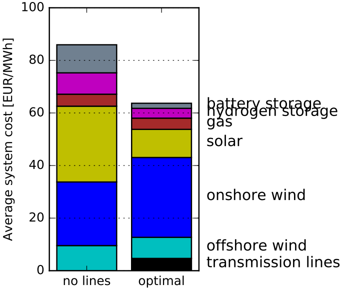


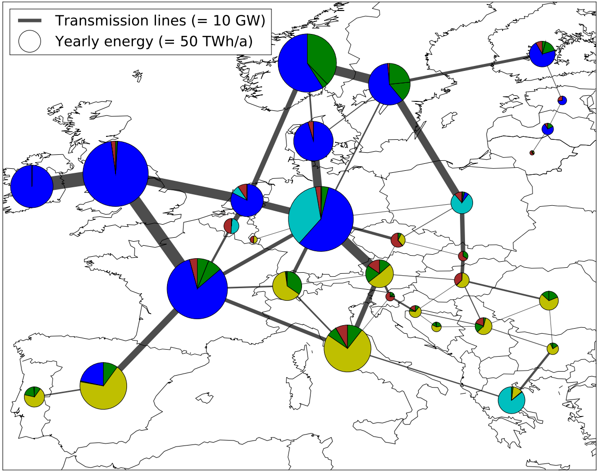
Costs: Cost-optimal expansion of interconnecting transmission

Technology by energy:



Average cost **€64/MWh**:

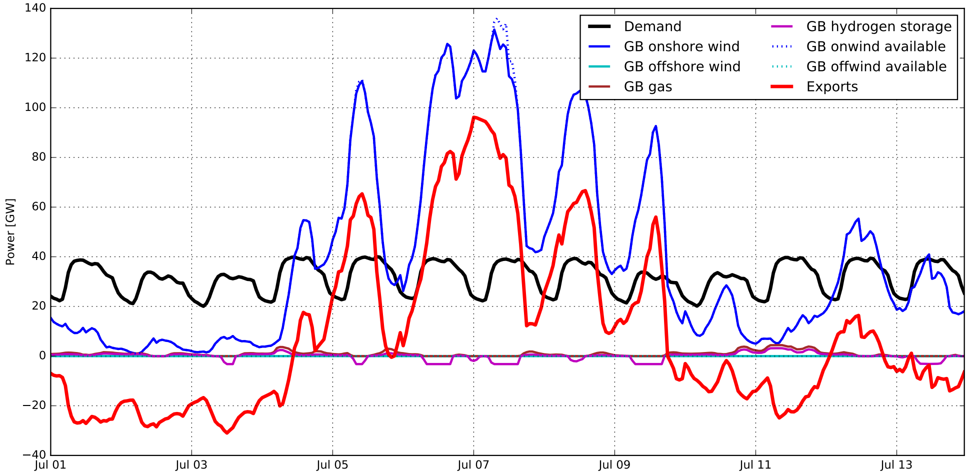




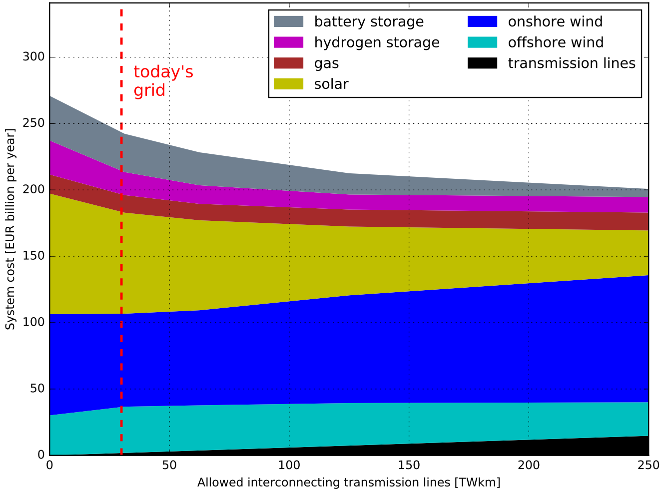
Large transmission expansion; onshore wind dominates. This optimal solution may run into public acceptance problems.

Dispatch with cost-optimal interconnecting transmission

Almost all excess wind can be now be exported:

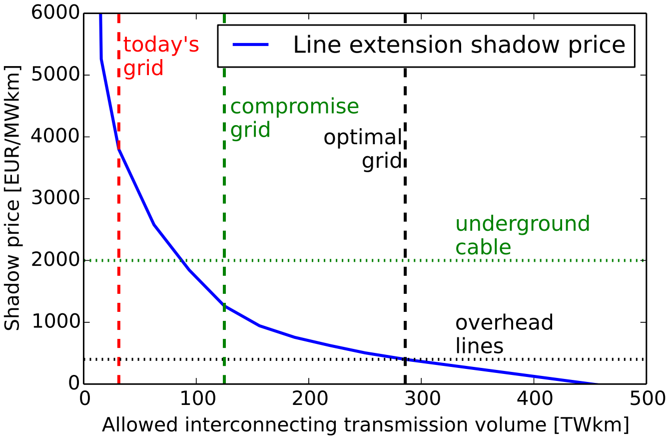


Electricity Only Costs Comparison



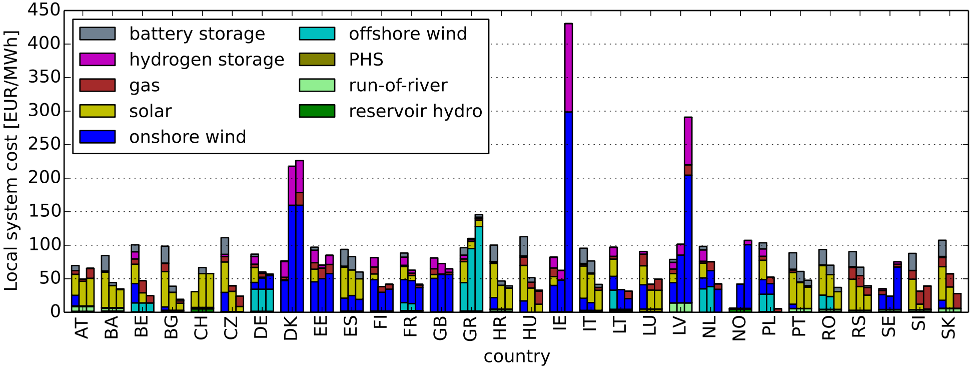
- Average total system costs can be as low as € 64/MWh
- Energy is dominated by wind (64% for the cost-optimal system), followed by hydro (15%) and solar (17%)
- Restricting transmission results in more storage to deal with variability, driving up the costs by up to 34%
- Many benefits already locked in at a few multiples of today’s grid

Grid expansion CAP shadow price μ_{trans} as CAP relaxed



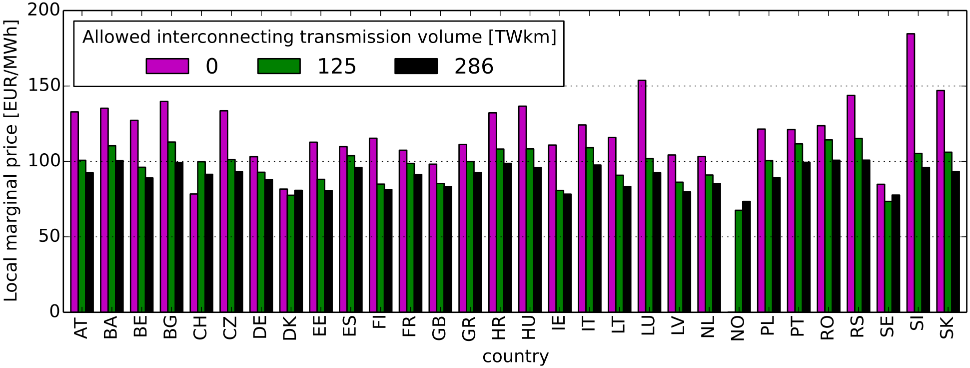
- With overhead lines the optimal system has around 7 times today's transmission volume
- With underground cables (5-8 times more expensive) the optimal system has around 3 times today's transmission volume

As transmission volumes increase, costs become more unequally distributed...

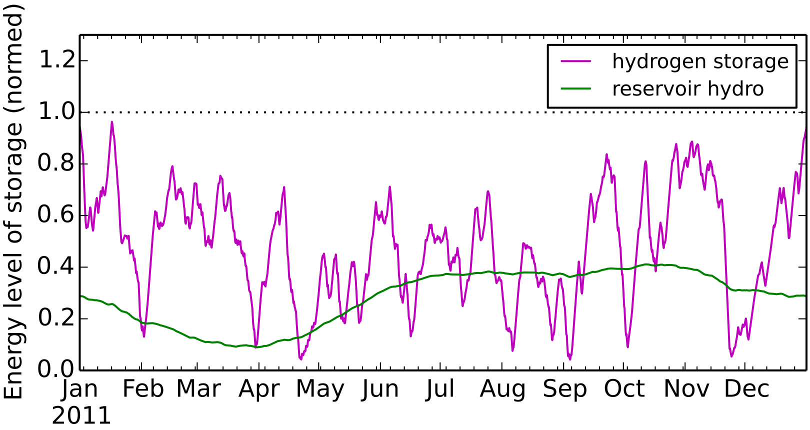


Distribution of prices

...while market prices converge.

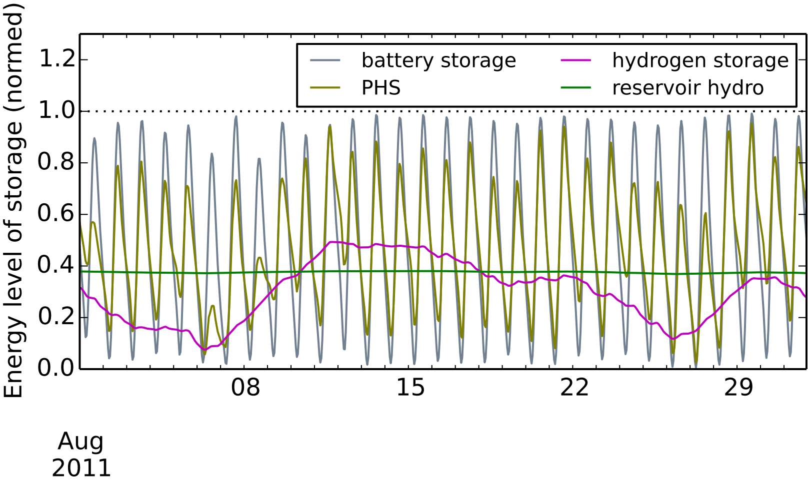


Different flexibility options have difference temporal scales



- Hydro reservoirs are **seasonal**
- Hydrogen storage is **synoptic** (i.e. weekly)

Different flexibility options have difference temporal scales



- Pumped hydro and battery storage are **daily**

You Try

For more details, see the paper **The Benefits of Cooperation** (2017).

The basic result (benefit of European interconnection versus national balancing) can also be seen using the online toy model:

https://model.energy/

Look at the differences of wind and solar feed-in and optimal storage solutions for:

- **City: Karlsruhe**
- **Country: Germany**
- **Continent: Europe**