Energy Systems, Summer Semester 2022
Lecture 11: Investment

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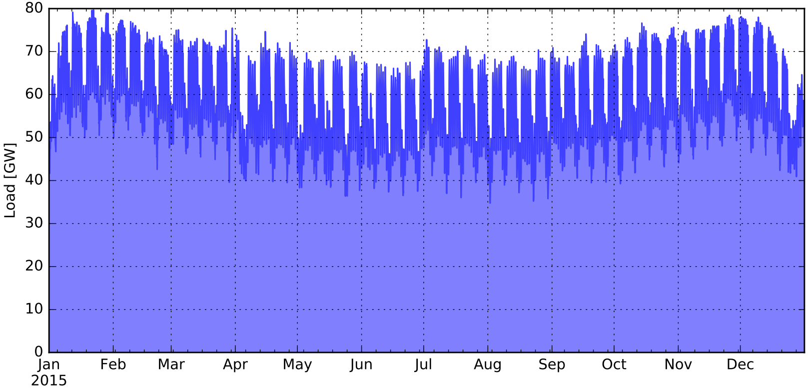
Duration Curves and Capacity Factors: Examples from Germany in 2015

Load curve

Here's the electrical demand (load) in Germany in 2015:

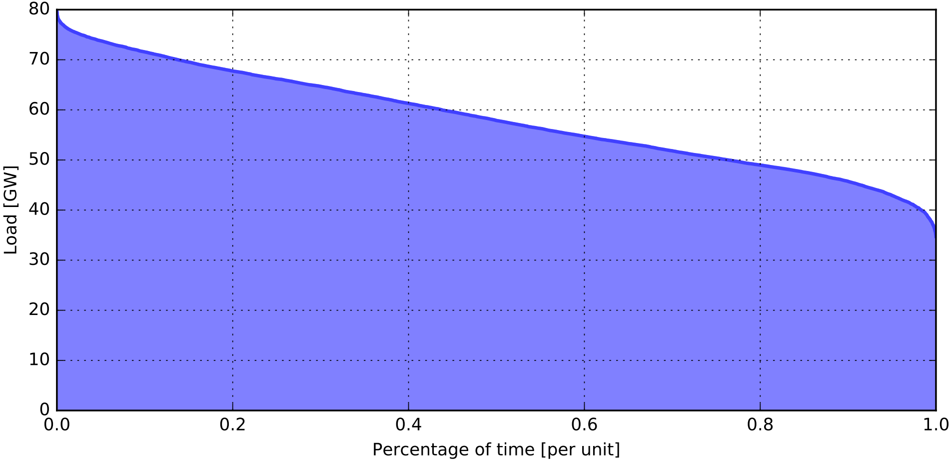


To understand this curve better and its implications for the market, it's useful to stack the hours of the year from left to right in order of the amount of load.

Load duration curve

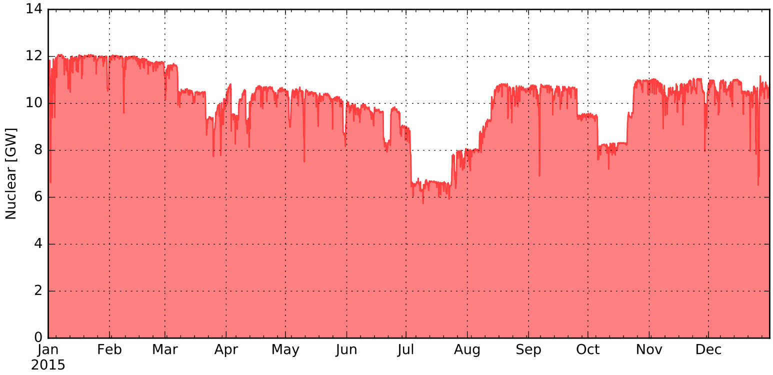
This re-ordering is called a **duration curve**.

For the load it's the **load duration curve**.



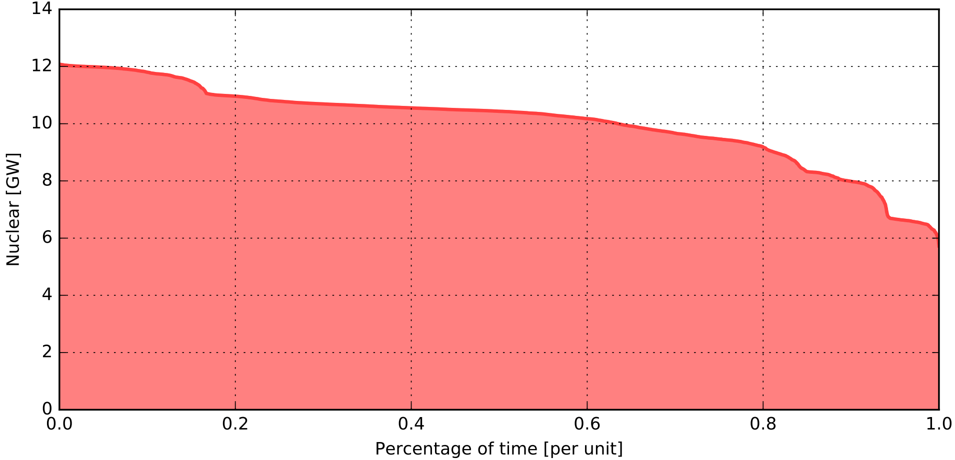
Nuclear curve

Can do the same for nuclear output:



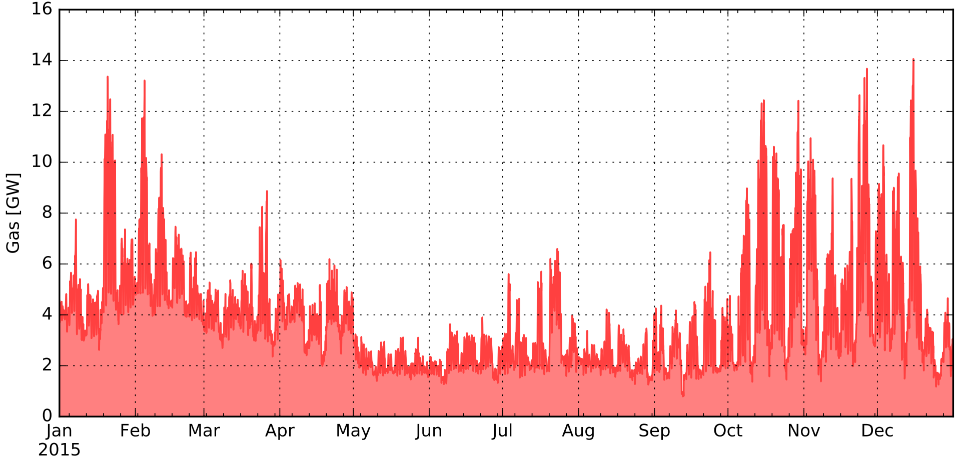
Nuclear duration curve

Duration curve is pretty flat, because it is economic to run nuclear almost all the time as **baseload plant**:



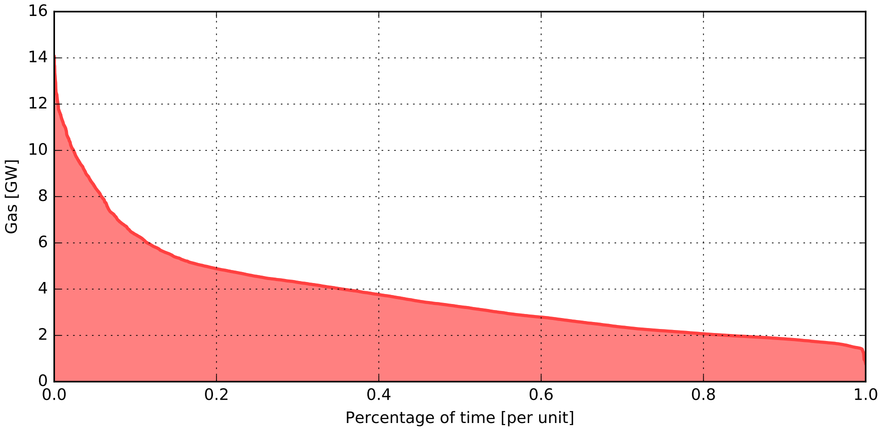
Gas curve

Can do the same for gas output:



Gas duration curve

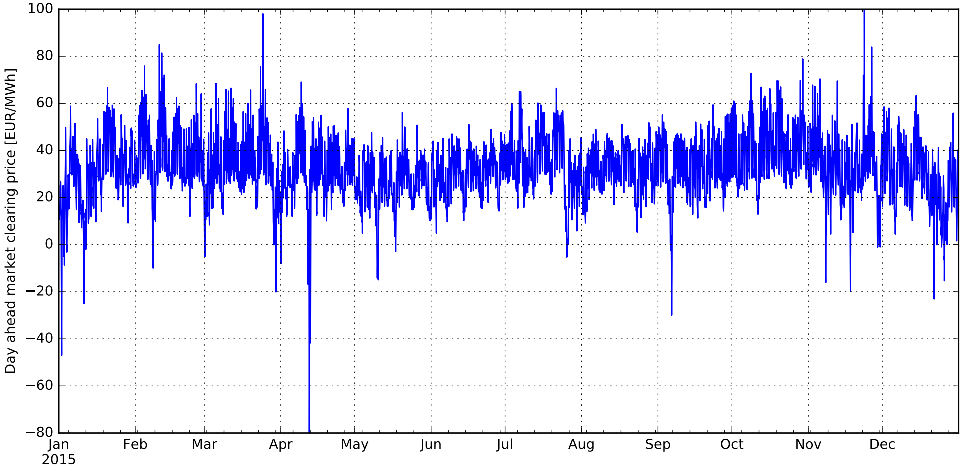
Duration curve is partially flat (for heat-driven CHP) and partially peaked (for **peaking plant**):



The capacity factor for gas is much lower - more like 20%.

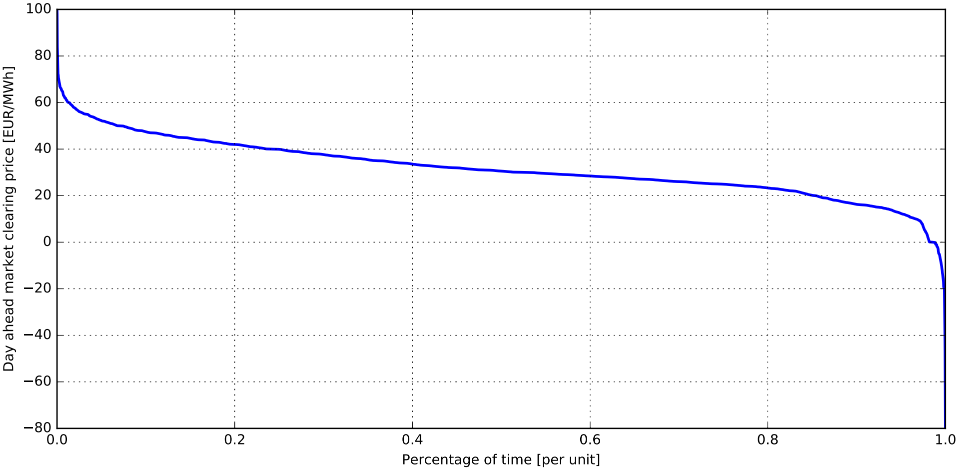
Price curve

Can do the same for price during the year:



Price duration curve

By ordering we get the **price duration curve**:



Question

Now we are in a position to consider the questions:

- What determines the distribution of investment in different generation technologies?
- How is it connected to variable costs, capital costs and capacity factors?

We will find the price and load duration curves very useful.

Investment Optimisation: Dispatchable Generation

Investment optimisation

Now we also optimise **investment** in the **capacities** of generators, storage and network lines for the **whole system** not just a single plant operator, to maximise **long-run efficiency**.

We will promote the capacities $G_{i,s}$, $G_{i,r,*}$, $E_{i,r}$ and F_ℓ to optimisation variables.

For generation investment, we want to answer the following questions:

- What determines the distribution of investment in different generation technologies?
- How is it connected to variable costs, capital costs and capacity factors?

We will find price and load duration curves very useful.

Definition of long-run efficiency

Up until now we have considered **short-run** equilibria that ensure **short-run** efficiency (static), i.e. they make the best use of presently available productive resources.

Long-run efficiency (dynamic) requires in addition the optimal investment in productive capacity.

Concretely: given a set of options, costs and constraints for different generators (nuclear/gas/wind/solar) what is the optimal generation portfolio for maximising long-run welfare?

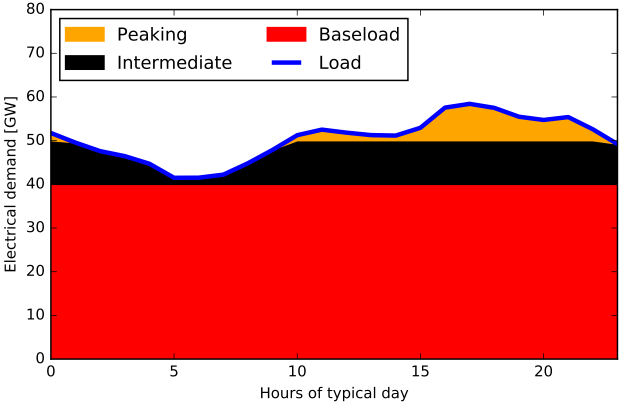
From an individual generators' perspective: how best should I invest in extra capacity?

We will show again that with perfect competition and no barriers to entry, the system-optimal situation can be reached by individuals following their own profit.

Baseload versus Peaking Plant

Load (= Electrical Demand) is low during night; in Northern Europe in the winter, the peak is in the evening.

To meet this load profile, cheap **baseload** generation runs the whole time; more expensive **peaking plant** covers the difference.



Source: Tom Brown

Different types of generators

Fuel/Prime mover	Marginal cost	Capital cost	Controllable	Predictable days ahead	CO2
Oil	V. High	Low	Yes	Yes	Medium
Gas OCGT	High	Low	Yes	Yes	Medium
Gas CCGT	Medium	Medium	Yes	Yes	Medium
Hard Coal	Medium	Lowish	Yes	Yes	High
Brown Coal	Low	Medium	Partly	Yes	High
Nuclear	V. Low	High	Partly	Yes	Zero
Hydro dam	Zero	High	Yes	Yes	Zero
Wind/Solar	Zero	High	Down	Partly	Zero

System-optimal dispatchable generator capacities and dispatch

Suppose we have generators labelled by s at a single node with **marginal costs** o_s for each unit of production $g_{s,t}$ and **specific capital costs** c_s for fixed costs regardless of the rate of production (e.g. investment in building capacity G_s). For a variety of demand values d_t that occur with probability p_t ($\sum_t p_t = 1$) we optimise the total **average hourly system costs**

$$\min_{\{g_{s,t}\},\{G_s\}} \left[\sum_s c_s G_s + \sum_{s,t} p_t o_s g_{s,t} \right]$$

such that (rescaling the KKT multipliers by p_t to simplify later formulae)

$$\begin{aligned} \sum_s g_{s,t} = d_t \qquad &\leftrightarrow \qquad p_t \lambda_t \\ -g_{s,t} \leq 0 \qquad &\leftrightarrow \qquad p_t \underline{\mu}_{s,t} \\ g_{s,t} - G_s \leq 0 \qquad &\leftrightarrow \qquad p_t \bar{\mu}_{s,t} \end{aligned}$$

Assume ordering $o_1 \leq o_2 \leq \cdots \leq o_S = v$ where $s = S$ is the generator for load-shedding, $o_S = v$ (Value of Lost Load), $c_S = 0$ (the capacity to shed load is assumed cost-free).

We've chosen the units here so that the total objective function has units €h^{-1}, the **average hourly system costs**.

$\sum_{s,t} p_t o_s g_{s,t}$ is the **expectation value** of the hourly production costs. $g_{s,t}$ has units MW, o_s has units €(MWh)$^{-1}$.

$c_s G_s$ is the investment cost averaged over each hour, i.e. the annualised investment cost $a(r, T)I_0$ (like a mortgage - we'll cover how to get this next lecture from the investment cost I_0, interest rate r and lifetime T via the annuity $a(r, T)$) divided by 8760, $\frac{a(r,T)I_0}{8760}$ (we can also add the fixed O&M costs B to it). G_s has units MW, c_s has units €MW^{-1}h^{-1}.

We could have instead optimised **average yearly system costs**, then $c_s G_s$ would simply be the annuity, and instead of weighting with p_t such that $\sum_t p_t = 1$, we replace it with a weighting w_t such that $\sum_t w_t = 8760$. In this case, the total objective would have units €a^{-1}.

Stationarity for $g_{s,t}$ gives us for each s and t the same equation we had without capacity optimisation:

$$0 = \frac{\partial \mathcal{L}}{\partial g_{s,t}} = p_t \left(o_s - \lambda^*_t - \bar{\mu}^*_{s,t} + \underline{\mu}^*_{s,t} \right)$$

and for the capacity G_s for each s it relates the capital cost c_s to the KKT multipliers for the capacity constraint:

$$0 = \frac{\partial \mathcal{L}}{\partial G_s} = c_s + \sum_t p_t \bar{\mu}^*_{s,t}$$

and from complementarity we get

$$\bar{\mu}^*_{s,t}(g^*_{s,t} - G^*_s) = 0$$
$$\underline{\mu}^*_{s,t} g^*_{s,t} = 0$$

and dual feasibility (negative for minimisation) $\bar{\mu}^*_{s,t}, \underline{\mu}^*_{s,t} \leq 0$.

The solution for the dispatch $g^*_{s,t}$ is exactly the same as without capacity optimisation. For each t, find the marginal generator m where the supply curve intersects with the demand d_t, i.e. the m where $\sum_{s=1}^{m-1} G^*_s < d_t < \sum_{s=1}^{m} G^*_s$.

The **marginal generator will set the price** $\lambda^*_t = o_m$, like before.

For $s < m$ we have $g^*_{s,t} = G^*_s$, $\underline{\mu}^*_{s,t} = 0$, $\bar{\mu}^*_{s,t} = o_s - \lambda^*_t \leq 0$.

For $s = m$ we have $g^*_{m,t} = d_t - \sum_{s=1}^{m-1} G^*_s$ to cover what's left of the demand. Since $0 < g^*_{m,t} < G_m$ we have $\underline{\mu}^*_{m,t} = \bar{\mu}^*_{m,t} = 0$ and therefore $\lambda^*_t = o_m$.

For $s > m$ we have $g^*_{s,t} = 0$, $\underline{\mu}^*_{s,t} = \lambda^*_t - o_s \leq 0$, $\bar{\mu}^*_{s,t} = 0$.

What about the G^*_s?

System-optimal generator capacities and dispatch

The G_s^* are determined implicitly based on the interactions between costs and prices.

From stationarity we had the relation

$$c_s = -\sum_t p_t \bar{\mu}_{s,t}^*$$

The $\bar{\mu}_{s,t}^*$ were only non-zero with $\lambda_t^* > o_s$ so we can re-write this as

$$c_s = \sum_{t|\lambda_t^* > o_s} p_t(\lambda_t^* - o_s)$$

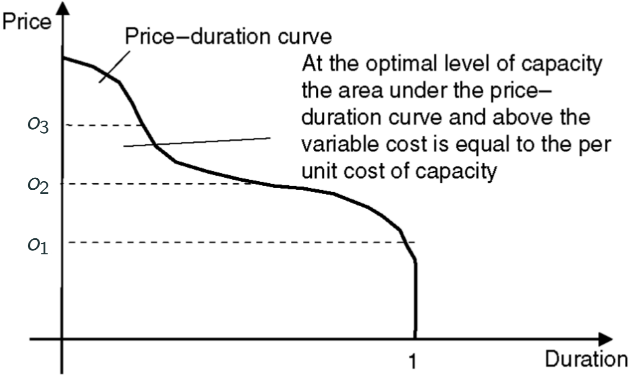
This is the **average inframarginal rent the generator makes in the short-run market**, which is its contribution towards covering its fixed costs.

'Increase capacity until marginal increase in profit equals the cost of extra capacity.'

Above this capacity the generator makes less money from the market than its cost $\Rightarrow$ bad investment.

Multiple price duration

The optimal mix of generation is where, for each generation type, the area under the price–duration curve and above the variable cost of that generation type is equal to the fixed cost of adding capacity of that generation type.



Source: Biggar and Hesamzadeh, 2014

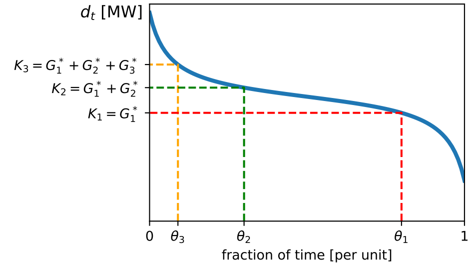
Assume again we have $o_1 \leq o_2 \leq \dots \leq o_S = v$ and $K_s = \sum_{p=1}^{s} G_p^*$ then:

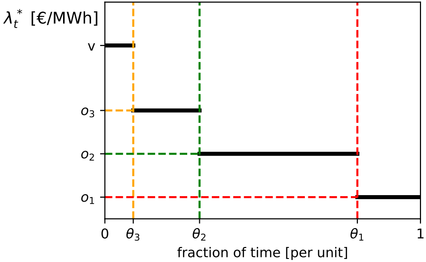
$$\lambda_t = \begin{cases} v & \text{for } d_t > K_{S-1} \\ o_s & \text{if } K_{s-1} < d_t \leq K_s, \end{cases} \qquad \text{for } s = 1, \dots S-1$$

Looking at the area under the price duration curve but above the variable cost, we then find:

$$c_s = (v - o_s)P(d > K_{S-1}) + \sum_{j=s+1}^{S-1} (o_j - o_s)P(K_{j-1} < d \leq K_j)$$

Example with three generators plus load shedding



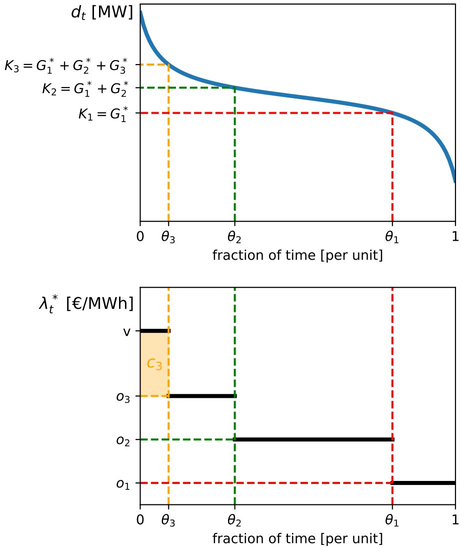


Example for $S = 4$ (3 generators plus load-shedding).

The upper graph is the load duration curve.
The y-axis is marked with the summed capacities of the generators $K_s = \sum_{p=1}^{s} G_p^*$. These meet the curve at $\theta_s = P(d_t > K_s)$ (the definition of load duration curve).

The lower graph is the price duration curve.
During the time when generator s is price-setting, the price is o_s.
When $d_t > G_1^* + G_2^* + G_3^*$ then we have load-shedding and the price is v.

Example with three generators plus load shedding

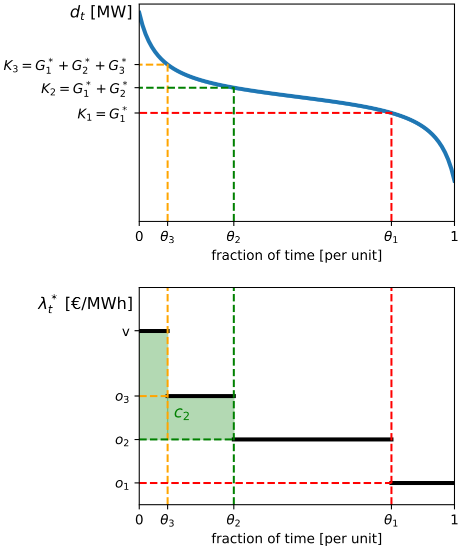


How is this related to the capital costs?

For generator 3, the capital cost c_3 is the area below the price duration curve when $\lambda_t^* > o_3$.
This only happens when there is load-shedding and $\lambda_t^* = v$ when $d_t > K_3 = G_1^* + G_2^* + G_3^*$.
The area is given by

$$
\begin{aligned}
c_3 &= (v - o_3)\theta_3 \\
&= (v - o_3)P(d_t > K_3)
\end{aligned}
$$

Example with three generators plus load shedding

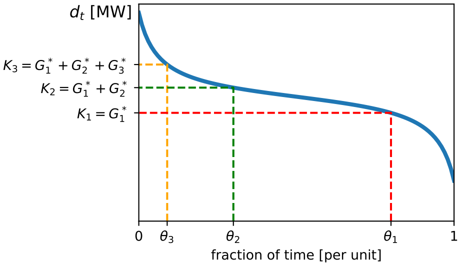


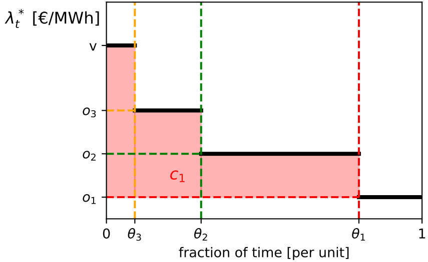
How is this related to the capital costs?

For generator 2, the capital cost c_2 is the area below the price duration curve when $\lambda_t^* > o_2$.
This only happens when there is load-shedding or when generator 3 is price-setting.
The area is given by

$$
\begin{aligned}
c_2 &= (v - o_2)\theta_3 + (o_3 - o_2)(\theta_2 - \theta_3) \\
&= (v - o_2)P(d_t > K_3) \\
&\qquad + (o_3 - o_2)\left(P(d_t > K_2) - P(d_t > K_3)\right) \\
&= (v - o_2)P(d_t > K_3) + (o_3 - o_2)P(K_2 < d_t \leq K_3)
\end{aligned}
$$

Example with three generators plus load shedding





How is this related to the capital costs?

Finally for generator 1, the capital cost c_1 is the area below the price duration curve when $\lambda_t^* > o_1$.
This only happens when there is load-shedding or when generator 2 or 3 is price-setting.
The area is given by

$$
\begin{aligned}
c_1 &= (v - o_1)\theta_3 + (o_3 - o_1)(\theta_2 - \theta_3) + (o_2 - o_1)(\theta_1 - \theta_2) \\
&= (v - o_1)P(d_t > K_3) \\
&\qquad + (o_3 - o_1)\left(P(d_t > K_2) - P(d_t > K_3)\right) \\
&\qquad + (o_2 - o_1)\left(P(d_t > K_1) - P(d_t > K_2)\right) \\
&= (v - o_1)P(d_t > K_3) + (o_3 - o_1)P(K_2 < d_t \leq K_3) \\
&\qquad + (o_2 - o_1)P(K_1 < d_t \leq K_2)
\end{aligned}
$$

These equations can be rewritten recursively using the substitution $\theta_s = P(d > K_s)$:

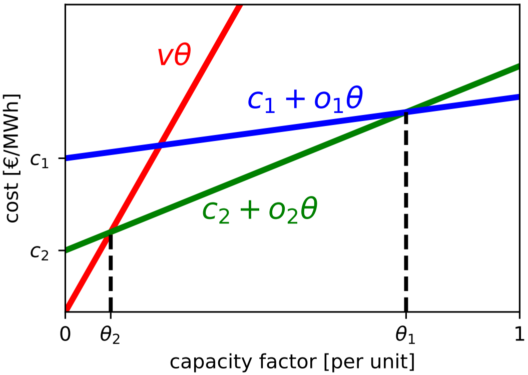
$$c_{S-1} + \theta_{S-1} o_{S-1} = v\theta_{S-1}$$
$$c_s + \theta_s o_s = c_{s+1} + \theta_s o_{s+1} \qquad \forall s = 1, \ldots S-2$$

The first equation can be solved to find θ_{S-1}, then the other equations can be solved recursively to find the remaining θ_s. The θ_s correspond to the optimal **capacity factors** of each type of generator, which correspond to the fraction of time the generator runs at full power.

Screening curve

The costs $c_s + o_s\theta$ as a function of the capacity factors θ can be drawn together as a **screening curve** (more expensive options are *screened* from the optimal inner polygon).

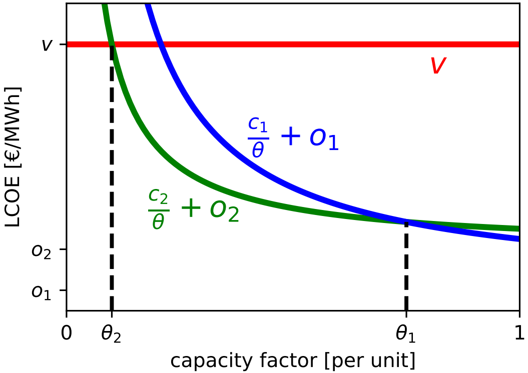
The intersection points determine which generators are optimal for which capacity factors.



$c_s + o_s\theta$ is the cost per MW and per hour of delivering power for θ of the time. c_s gives the intercept of the y axis; o_s gives the slope.

In this example we have load shedding, a baseload generator 1 and a peaking generator 2.
For a capacity factor between 0 and θ_2, it is cheapest to shed load.
Between θ_2 and θ_1 the peaking generator 2 is lowest cost.
Above θ_1 the baseload generator 1 is best.

Relation to levelised cost of electricity (LCOE)

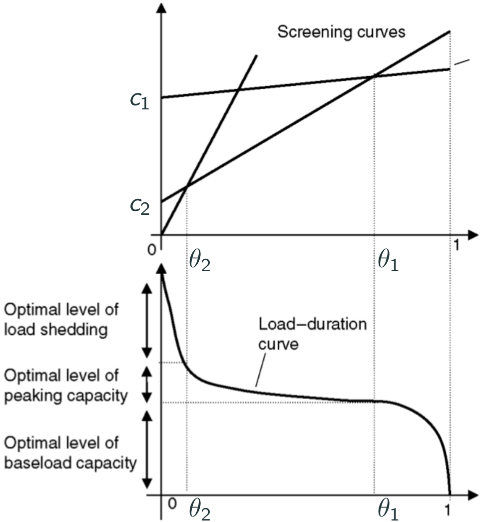


- $c_s + o_s\theta$ is the cost per MW and per hour of delivering power for θ of the time.
- To get the **levelised cost of electricity (LCOE)**, the cost per delivered energy, we divide by θ:

$$\mathrm{LCOE}_s = \frac{c_s}{\theta} + o_s$$

- The intersection points θ_s are the same, but it's now harder to read the graph.
- For peaking generator 2 with low capital cost c_2 and high marginal cost o_2, $\mathrm{LCOE}_2 \to o_2$ as $\theta \to 1$.

Screening curve versus Load duration



The screening curve allows us to read of the optimal generator capacities G_p^* from the load duration curve.

- We match the intersection points θ_s to the load duration curve.
- The values of the load duration curve at θ_s tell us what the cumulative sums $K_s = \sum_{p=1}^{s} G_p^*$ of the generator capacities are.
- This allows us to deduce the generator capacities G_p^*.

Source: Biggar and Hesamzadeh, 2014

Example: 2 generation technologies and load shedding

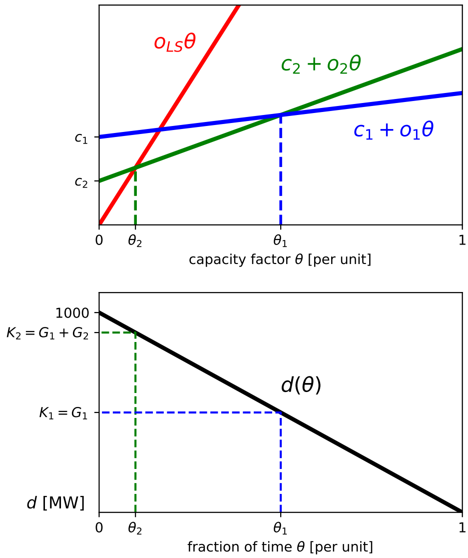
Suppose that electrical demand is perfectly inelastic with a load duration curve given by $d(\theta) = 1000 - 1000\theta$ for $0 \leq \theta \leq 1$. Suppose that there are two different types of generation with variable costs of 2 and 12 €/MWh respectively, together with load-shedding at a cost of 1012 €/MWh. The fixed costs of the two generation types are 15 and 10 €/MWh respectively. See the below table for a summary of the costs.

Generator	o_s [€/MWh]	c_s [€/MW/h]
1	2	15
2	12	10
LS	1012	0

Example: 2 generation technologies and load shedding

1. What is the interpretation of the load duration curve?
2. Below which capacity factor θ_1 is it cheaper to run Generator 2 rather than to run Generator 1?
3. Below which capacity factor θ_2 is it cheaper to shed load than to run Generator 2?
4. Plot the costs as a function of θ and mark these intersection points on a screening curve.
5. Find the optimal capacities of Generators 1 and 2 in this market.

Example: 2 generation technologies and load shedding



Procedure:

- Draw the screening curve and load duration curve $d(\theta)$.
- Determine the intersection points θ_s from the screening curve.
- Compute the cumulative generator capacity sums from the load duration curve $d(\theta_s) = K_s = \sum_{p=1}^{s} G_p^*$.
- Find the capacities G_s^* from the cumulative sums K_s.

Example: 2 generation technologies and load shedding

To find θ_1, solve for the intersection of Generator 1's cost curve with Generator 2's cost curve as a function of capacity factor:

$$c_1 + \theta_1 o_1 = c_2 + \theta_1 o_2 \qquad \Rightarrow \qquad 15 + 2\theta_1 = 10 + 12\theta_1$$

This gives $\theta_1 = 0.5$. At this point the demand is $d(0.5) = 500$ MW therefore

$$K_1 = G_1^* = 500 \text{ MW}$$

To find θ_2, solver for where Generator 2 crosses the load-shedding line:

$$c_2 + \theta_2 o_2 = c_{LS} + \theta_2 o_{LS} \qquad \Rightarrow \qquad 10 + 12\theta_2 = 1012\theta_2$$

This gives $\theta_2 = 0.01$. At this point the demand is $d(0.01) = 990$ MW so:

$$K_2 = G_1^* + G_2^* = 990 \text{ MW}$$

i.e. $G_2^* = 490$ MW. The remaining load is shed, $G_{LS}^* = 10$ MW.

Investment Optimisation: Transmission

Investment optimisation: transmission

As before, our approach to the question of **"What is the optimal amount of transmission"** is determined by the most efficient long-term solution. Promote F_ℓ to an optimisation variable with specific capital cost c_ℓ. For nodes i and transmission lines ℓ enforcing KCL but not KVL:

$$\min_{\{g_{i,s,t}\},\{G_{i,s}\},\{f_{\ell,t}\},\{F_\ell\}} \left[\sum_s c_{i,s} G_{i,s} + \sum_{i,s,t} p_t o_{i,s} g_{i,s,t} + \sum_\ell c_\ell F_\ell\right]$$

such that

$$\sum_s g_{i,s,t} - \sum_\ell K_{i\ell} f_{\ell,t} = d_{i,t} \quad \leftrightarrow \quad p_t \lambda_{i,t}$$

$$-g_{i,s,t} \leq 0 \quad \leftrightarrow \quad p_t \underline{\mu}_{i,s,t}$$

$$g_{i,s,t} - G_{i,s} \leq 0 \quad \leftrightarrow \quad p_t \bar{\mu}_{i,s,t}$$

$$f_{\ell,t} - F_\ell \leq 0 \quad \leftrightarrow \quad p_t \bar{\mu}_{\ell,t}$$

$$-f_{\ell,t} - F_\ell \leq 0 \quad \leftrightarrow \quad p_t \underline{\mu}_{\ell,t}$$

Investment optimisation: transmission

From stationarity for $f_{\ell,t}$ we find

$$0 = \frac{\partial \mathcal{L}}{\partial f_{\ell,t}} = p_t \left(\sum_i K_{i\ell} \lambda^*_{i,t} - \bar{\mu}^*_{\ell,t} + \underline{\mu}^*_{\ell,t} \right)$$

I.e. the KKT multipliers $\bar{\mu}^*_{\ell,t}$ or $\underline{\mu}^*_{\ell,t}$ are non-zero when the line ℓ is congested (by definition), at which time one of them equals the price difference between the ends of the line.

For the investment we find from stationarity $0 = \frac{\partial \mathcal{L}}{\partial F_\ell}$

$$c_\ell = -\sum_t p_t \left(\bar{\mu}^*_{\ell,t} + \underline{\mu}^*_{\ell,t} \right)$$

Remember that $\bar{\mu}^*_{\ell,t}$ and $\underline{\mu}^*_{\ell,t}$ are only non-zero when the line is congested.

Exactly as with generation dispatch and investment, we continue to invest in transmission until the marginal benefit of extra transmission (i.e. extra congestion rent for extra capacity) is equal to the marginal cost of extra transmission. This determines the optimal investment level.