Energy Economics, Winter Semester 2026-7
Lecture 3: Basics of Microeconomics

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Types of Markets

The role of markets

A **market** is where a group of sellers/producers and a group of buyers/consumers of a particular good or service come together for exchange.

Wherever there is **scarcity** we have an **allocation** question: who should consume, and who should produce?

And at what **price**?

Economists measure **efficiency** based on how well an allocation maximises **economic welfare**.

Monopoly versus perfect competition



Monopoly versus perfect competition

For **perfect competition** to apply we need:

- **many buyers and many sellers** so nobody can influence the price, i.e. all actors are **price takers**
- goods are exactly the same, i.e. **homogeneous goods**
- all actors have **perfect information**
- **no entry or exit barriers**

In a **monopoly** there is a **single seller**:

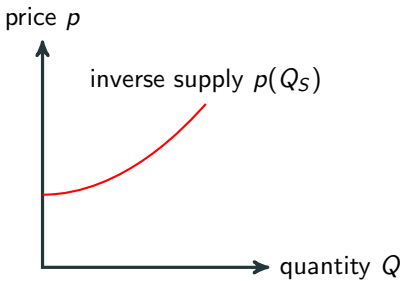
- Seller is **sole producer** and can influence the price of its output
- Seller has **market power**: the ability to maintain a price above the price under competition

Perfect Competition

Inverse supply functions

Each producing firm has its own **inverse supply function** which shows at which price p it is worthwhile to sell a given quantity Q_S. It indicates the **marginal cost** of the next unit of production at a given production level of Q_S units. Generally: the higher the quantity, the higher the price.

(It's the *inverse* because the supply function is the quantity as a function of price $Q_S(p)$.)



Inverse supply function: example

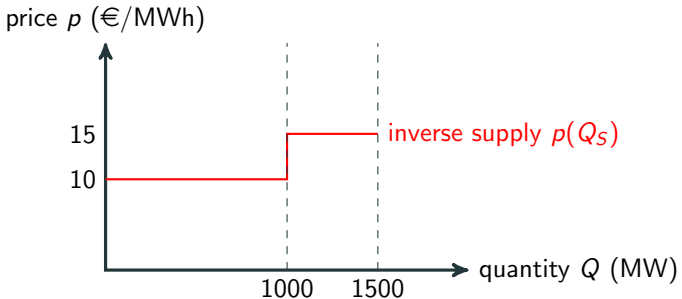
A generation firm has two coal plants. The new plant can produce 1000 MW at a cost of 10 €/MWh. An older less efficient plant can produce only 500 MW at a cost of 15 €/MWh.

What does the supply curve look like?

Inverse supply function: example

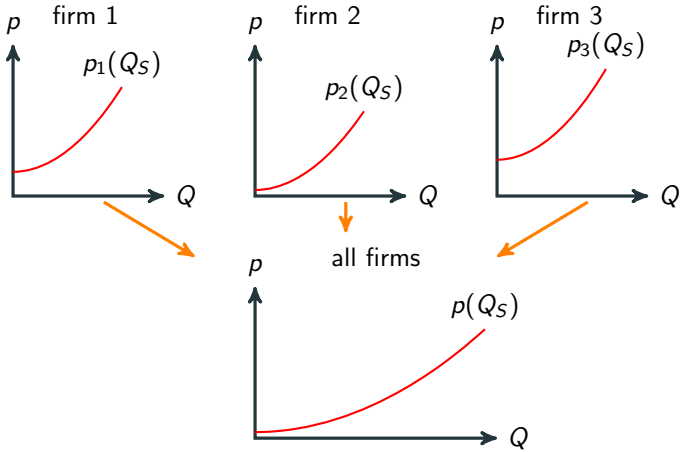
A generation firm has two coal plants. The new plant can produce 1000 MW at a cost of 10 €/MWh. An older less efficient plant can produce only 500 MW at a cost of 15 €/MWh.

What does the supply curve look like?



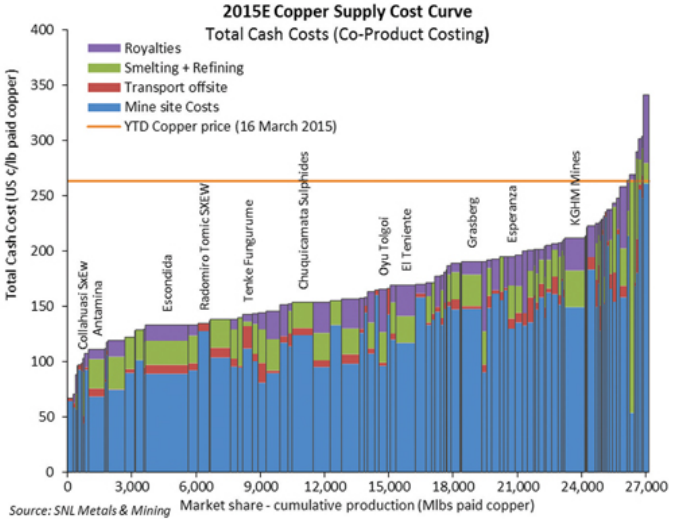
Aggregation of inverse supply functions

If many firms are active, we can sort and aggregate their inverse supply functions into a single supply function for the whole market.



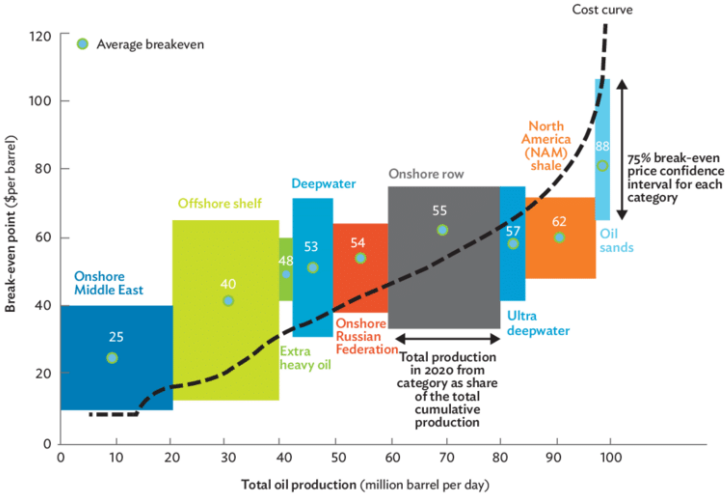
Example 1: Worldwide aggregated supply curve for copper

Copper is an important component of electrical machinery and distribution.



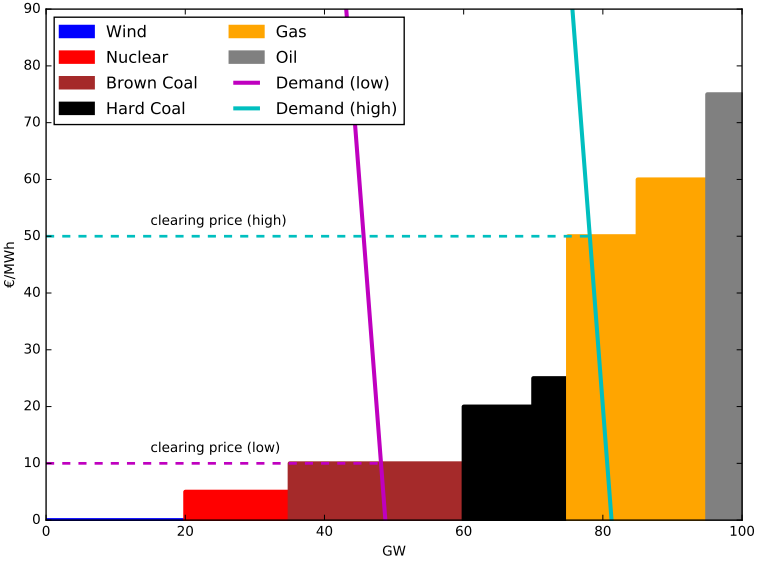
Example 2: Worldwide aggregated supply function for oil

Boxes indicate ranges for each supply source of oil.



Source: Rystad Energy (2015).

Example 3: German aggregated supply function for electricity



Supply side: direct versus opportunity costs

The price at which a seller is willing to sell their goods is determined by their costs of production:

- **Explicit, direct costs**: out-of-pocket expenses, money actually paid e.g. for materials, energy
- **Opportunity costs**: fictive costs representing the potential benefit or income that is foregone as a result of selecting one alternative over another when there is scarcity

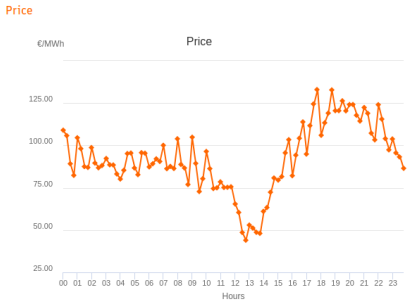
Examples:

- **Scarce time**: Going to cinema instead of working: direct cost is ticket 10 €, opportunity cost is 40 € one could have earned by working instead; total cost: 50 €
- **Scarce money**: Investing in rooftop solar instead of savings account: direct cost is price of solar panel, opportunity cost is the interest you could have earned; have to factor interest in comparison

Opportunity costs example: reservoir hydroelectricity

A hydroelectric dam has enough water stored to feed into the grid for one hour (**scarce water**). There are no fuel costs or other direct marginal costs. So you might think we should feed in at midday when the electricity price is 50 €/MWh. However, then we miss the chance to feed in later when the price is 125 €/MWh. These are the **opportunity costs** of foregoing later feed-in, which we have to take into account in our production costs. Once included, one would bid with marginal costs of 125 €/MWh, which makes sure we only feed-in at the best time in the evening.

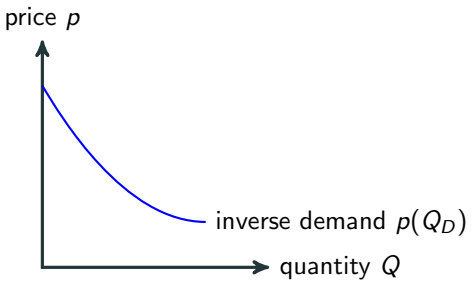




Inverse demand functions

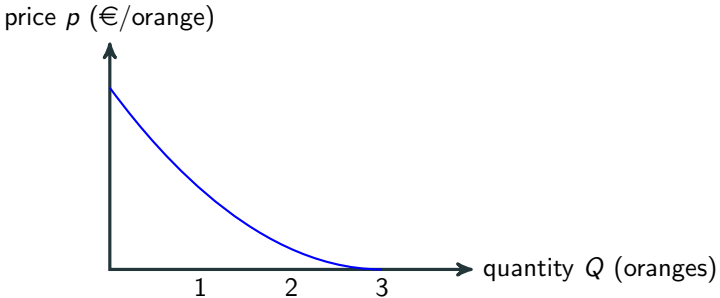
Each consumer has its own **inverse demand function** which shows which price p it is prepared to pay for a given quantity Q_D. It indicates the **willingness to pay** or **marginal utility** of the next unit of consumption at a given consumption level of Q_D units. Generally: the higher the quantity, the lower the price.

(It's the *inverse* because the demand function is the quantity as a function of price $Q_D(p)$.)



Inverse demand function: example

After sport I'm hungry and thirsty. I visit a stand selling oranges. The first orange is worth a lot to me. It partially satisfies my hunger and thirst, so the second is worth less to me. By the 3rd orange, I'm full.



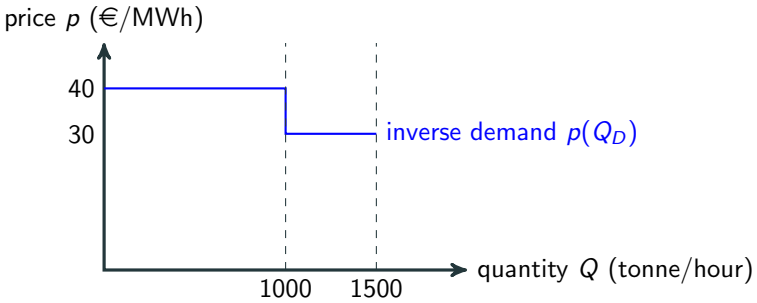
Inverse demand: example

Electrolysis is used to produce aluminium from bauxite ore. On the open market aluminium sells for €1200/tonne. The cost of materials is €600/tonne, which leaves €600/tonne to pay for electricity. A firm has two electrolysis units. A newer efficient one with capacity 1000 tonne/hour consumes 15 MWh/tonne, while an older inefficient one with capacity 500 tonne/hour consumes 20 MWh/tonne. How much are they willing to pay for electricity?

Inverse demand: example

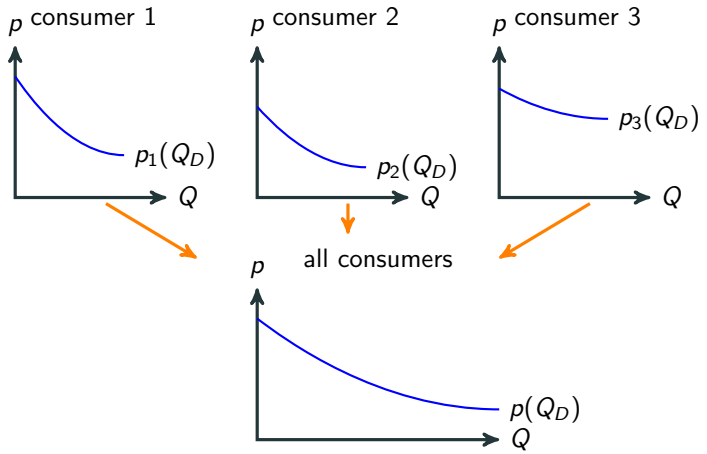
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New: $\frac{600€/\mathrm{t}}{15\mathrm{MWh/t}} = 40€/\mathrm{MWh}$, old $\frac{600€/\mathrm{t}}{20\mathrm{MWh/t}} = 30€/\mathrm{MWh}$.



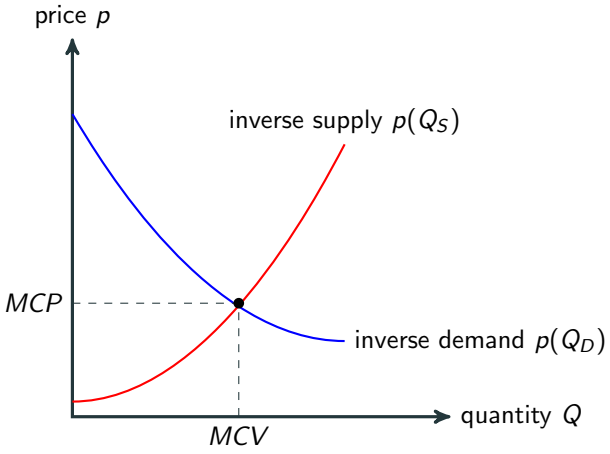
Aggregation of inverse demand functions

If many consumers are active, we can sort and aggregate their inverse demand functions into a single demand function for the whole market.

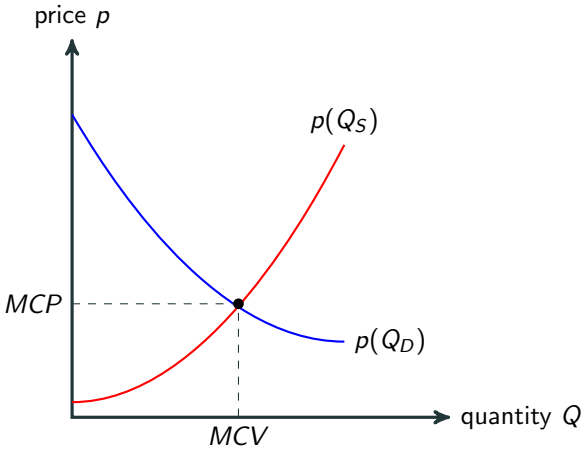


Market clearing price and volume

In a competitive market the **market clearing price** (MCP) and the **market clearing volume** (MCV) are set by the intersection of the inverse supply and demand functions.



Market clearing price and volume: important properties



- Every actor sees the same price.
- The price arises **decentrally** from the interaction of supply and demand curves.
- The same amount is supplied as is consumed.
- Since there are many suppliers and consumers, no single actor can influence the price (they are all **price takers**).
- The price is higher than the costs of each supplier whose offer is taken.
- The price is lower than the willingness to pay of each consumer whose bid is taken.

Market clearing price and volume: simple example

Consider a market with:

Aggregated inverse demand curve $p_D(Q_D) = 28 - 4Q_D$.

Aggregated inverse supply curve $p_S(Q_S) = 1 + 5Q_S$.

Q: What is the market clearing price and volume?

Solve

$$28 - 4Q = 1 + 5Q$$

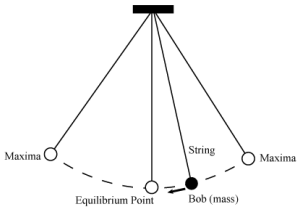
Find $Q^* = 3, p^* = 16$.

Price finding as an equilibrium

The price finding mechanism is often called an **equilibrium** in analogy to physics because the 'forces' of supply and demand are equal.

In physics, a **stable equilibrium** (like a pendulum) has negative feedback that returns the system to equilibrium after a disturbance. An **unstable equilibrium** (like a pencil on its end) would not return to its equilibrium.

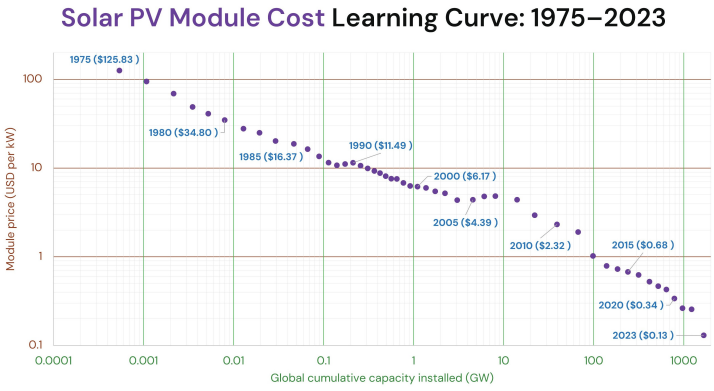
A market equilibrium is **stable** because the market actors have no incentive to change their behaviour, since they are already optimally adapted to the relevant market data.





The role of competition

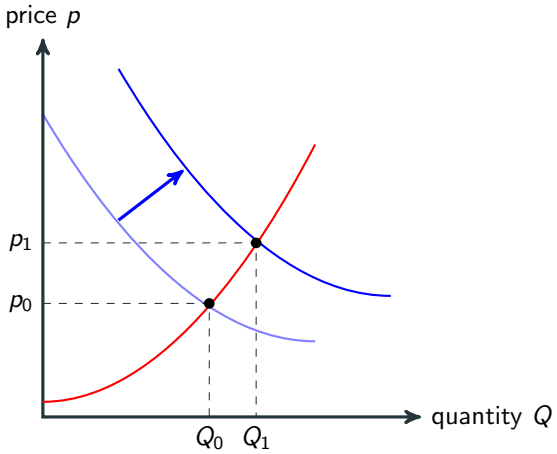
Competition for the favour of consumers creates **incentives to reduce prices and improve products**. Otherwise producers would not sell their products. Firms must constantly improve, creating **incentives for innovation**. Competition can unleash world-changing dynamics.



Ember. (2024). *Global Electricity Review 2024.*

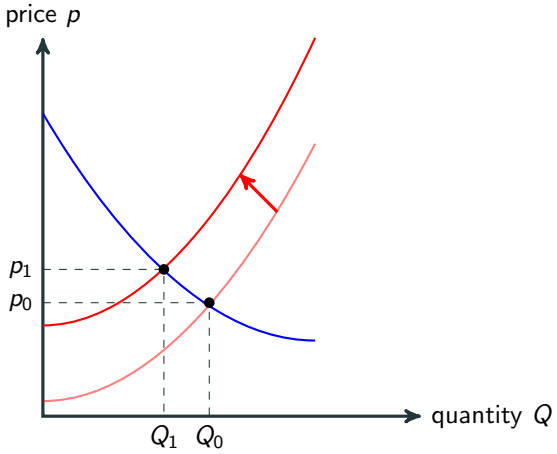
Reaction to increase in demand

If demand increases in volume and/or willingness to pay, the price goes up.



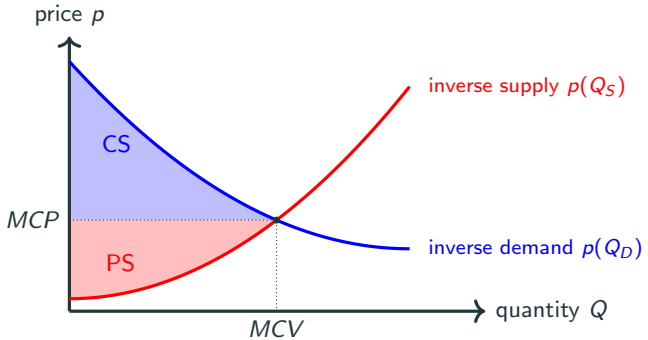
Reaction to increase in supply cost

If supply becomes more expensive or decreases in volume, the price goes up.



Consumer and producer surpluses

The **consumer surplus (CS)** is the total amount consumers are willing to pay minus what they actually pay. The **producer surplus (PS)** is the total revenue for producers minus their actual costs. The **total surplus** or **total welfare** $TW = CS + PS$ indicates the degree of efficiency of resource allocation.



Consumer and producer surpluses: example

Consider the previous example:

Aggregated inverse demand curve $p_D(Q_D) = 28 - 4Q_D$.

Aggregated inverse supply curve $p_S(Q_S) = 1 + 5Q_S$.

MCV and MCP are $Q^* = 3, p^* = 16$.

Q: What are the consumer and producer surpluses?

Consumer surplus: $CS = 0.5 * 3 * (28 - 16) = 18$.

Producer surplus: $PS = 0.5 * 3 * (16 - 1) = 22.5$.

Total welfare: $TW = CS + PS = 18 + 22.5 = 40.5$.

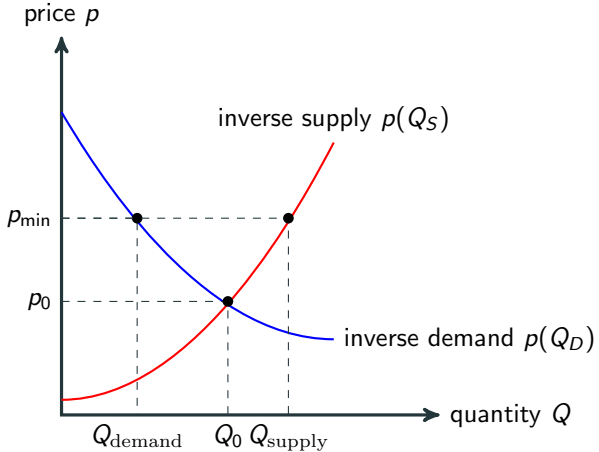
Market failures

Markets can fail to deliver welfare-maximisation if the conditions for perfect competition are not met. This can justify **state intervention**. Examples of **market failures** include:

- **Monopoly/monopsony**: Single actors dominate and use **market power** to set prices
- **Asymmetric information**: Some actors have more information than others (e.g. insider trading, used car salesman deliberately selling faulty goods)
- **Inhomogeneous goods**: Quality differences (e.g. fake Gucci handbags)
- **Market entry/exit barriers**: Examples include taxi cartels, notaries
- **Externalities ignored**: Costs are induced on third parties not active in the market, which are not priced in (e.g. air pollution, greenhouse gases)
- **Public goods**: Everyone benefits from them, but nobody has an incentive to produce them - **free-rider problem** (e.g. peace, biodiversity, dikes, education)

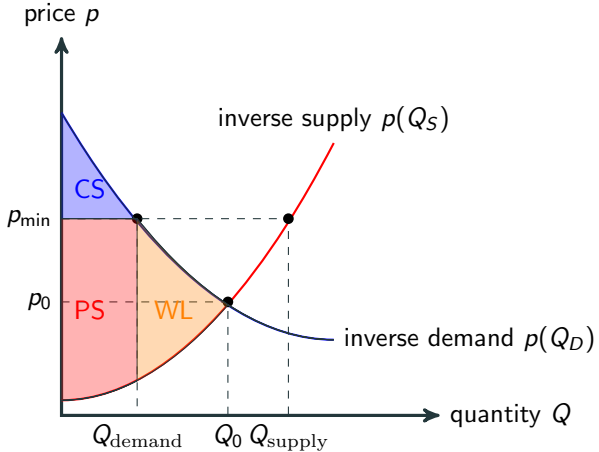
Reaction to state regulation: minimum price

A **minimum price** or **floor price** can lead to lower demand and/or excess supply (if there is a buyer of last resort like the state). Cf. butter mountains and milk lakes in EU in past; minimum wages.



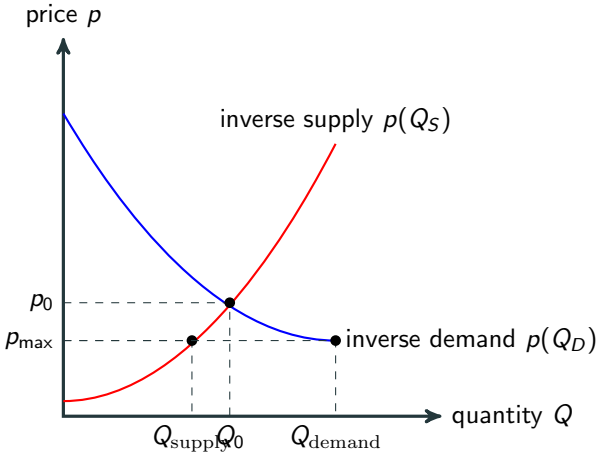
Minimum price: effect on total welfare

The introduction of a minimum price will **reduce** total welfare in this market, although it may have benefits in other parts of society (e.g. economic security for farmers and a living wage for workers). The **welfare loss (WL)** or **deadweight loss** corresponds to the orange triangle.



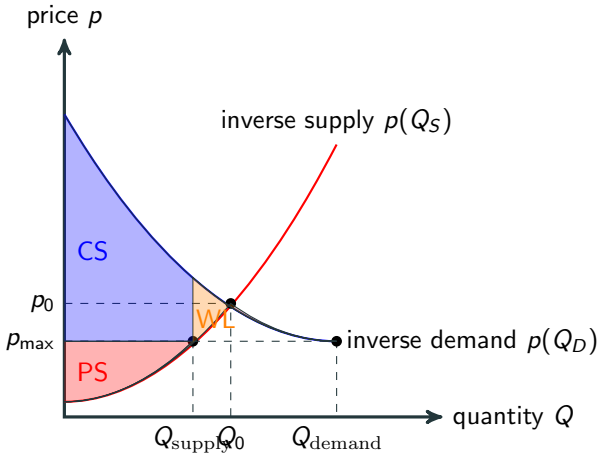
Reaction to state regulation: maximum price

A **maximum price** or **price cap** can lead to lower supply and/or excess demand. Cf. rent caps, energy price caps.



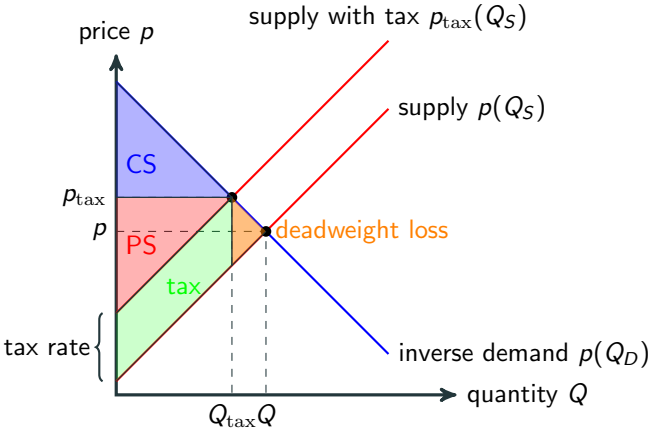
Maximum price: effect on total welfare

The introduction of a maximum price will **reduce** total welfare in this market, although it may have benefits in other parts of society (e.g. benefits for consumers). The **welfare loss (WL)** or **deadweight loss** corresponds to the orange triangle.



Consumer and producer surpluses: response to tax

A **tax** leads to **tax revenue** for the state, which counts towards the total welfare. However there is a **welfare loss** or **deadweight loss** compared to the case without the tax.



Market clearing price and volume with tax: simple example

Consider a market with:

Aggregated inverse demand curve $p_D(Q_D) = 28 - 4Q_D$.

Aggregated inverse supply curve $p_S(Q_S) = 1 + 5Q_S$.

Suppose we tax the product at a rate of 9€/unit. What is the market clearing price and volume now?

Solve

$$28 - 4Q = 10 + 5Q$$

Find $Q^* = 2, p^* = 20$.

Market clearing price and volume with tax: simple example

Consider a market with:

Aggregated inverse demand curve $p_D(Q_D) = 28 - 4Q_D$.

Aggregated inverse supply curve $p_S(Q_S) = 1 + 5Q_S$.

Suppose we tax the product at a rate of 9€/unit. How do the surpluses change?

Consumer surplus: $CS_{\text{tax}} = 0.5 * 2 * (28 - 20) = 8$.

Producer surplus: $PS_{\text{tax}} = 0.5 * 2 * (20 - 10) = 10$.

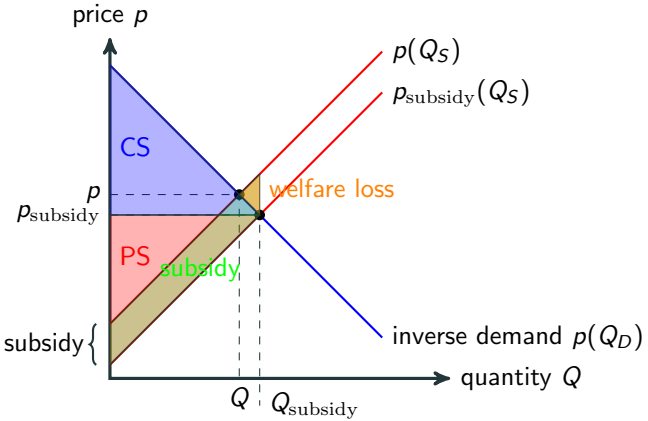
Total tax revenue: $TR_{\text{tax}} = 2 * 9 = 18$.

Total welfare includes tax revenue: $TW_{\text{tax}} = CS + PS + TR = 36$.

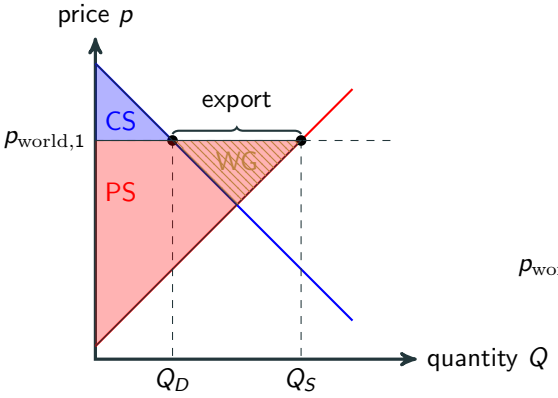
Deadweight loss: $TW - TW_{\text{tax}} = 40.5 - 18 - 18 = 4.5$.

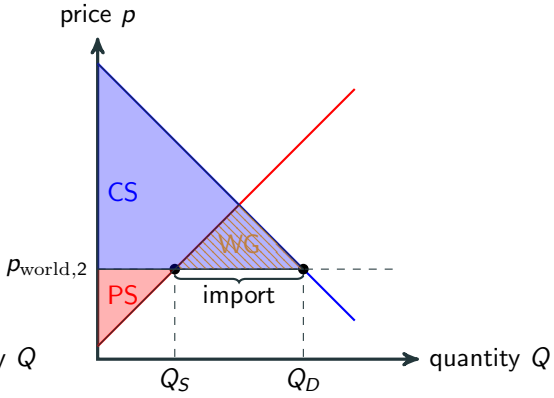
Consumer and producer surpluses: response to a subsidy

A **subsidy** leads to **tax burden** for the state (subsidy $\times$ Q_{subsidy}), which subtracts from the total welfare. There is still a **welfare loss** compared to the case without the subsidy.



Trade between 2 regions always leads to a **welfare gain** (WG) in each country, but can have a strong influence on the distribution between the consumer and producer surpluses.





Elasticity

Price elasticity

Elasticity is a is a measure of how much buyers and sellers respond to changes in market conditions.

Price elasticity measures the response to price changes. Price elasticity of demand $\eta_{p,Q}$ is a measure of how much the quantity consumed of a good responds to a change in the price of that good.

$$\text{Price elasticity of demand} = \frac{\%\text{ change in quantity consumed}}{\%\text{ change in price}}$$

$$\eta_{p,Q} = \frac{\frac{dQ}{Q}}{\frac{dp}{p}} = \frac{dQ}{dp} \cdot \frac{p}{Q}$$

Elastic versus inelastic demand

Demand-price elasticity is generally negative: the higher the price, the lower the demand. The magnitude of the elasticity allows it to be classified:

$-\infty < \eta_{p,Q} \leq -1$ **elastic demand** (big change in demand for small change in price)

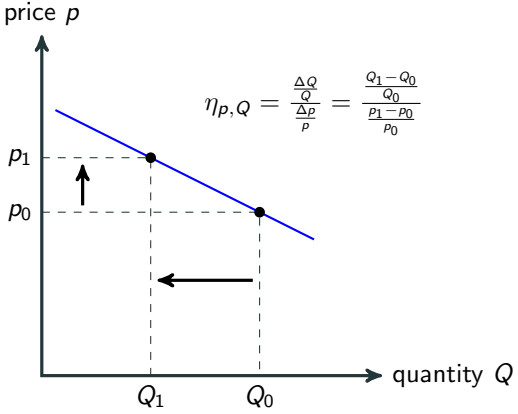
$\eta_{p,Q} = -1$ **isoelastic demand** (same % change in demand for % change in price)

$-1 < \eta_{p,Q} \leq 0$ **inelastic demand** (small change in demand for large change in price)

If demand does not respond at all to prices, $\eta_{p,Q} = 0$, it is **perfectly inelastic**.

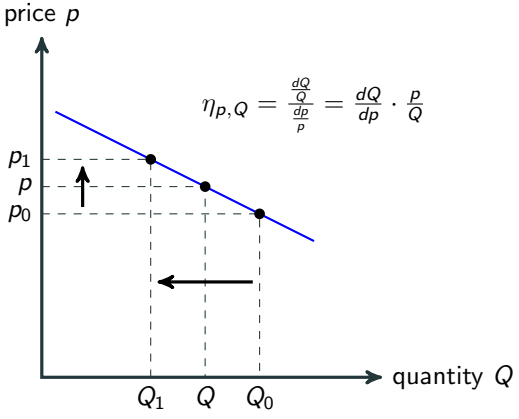
Arc elasticity

The **arc elasticity** can be measured if you know how the demand responds to a specific price increase from p_0 to p_1.



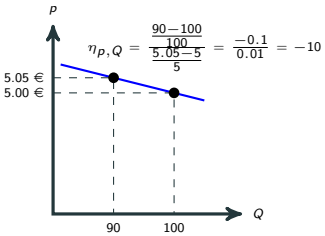
Point elasticity

The **point elasticity** is the infinitesimal version if you can precisely calculate the derivative.

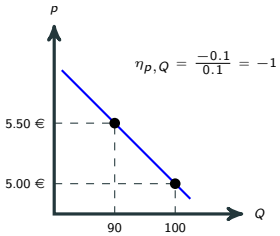


Examples of price-demand elasticity

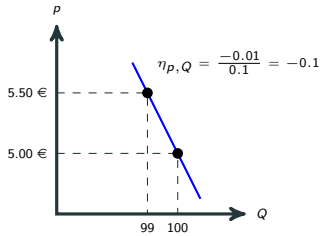
Consider the following arc elasticities:



very elastic (small change in price ⇒ big change in demand)



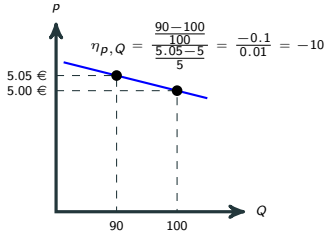
isoelastic (change in price ⇒ propotional change in demand)



very inelastic (large change in price ⇒ small change in demand)

Maximising revenue

Suppose a supplier has the market power to manipulate the price, and they have no marginal costs. How to maximise the **revenue** $R(p) = p \cdot Q(p)$? Consider effect of price increase:

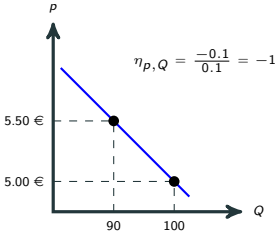


elastic

$R(p) = 5 \cdot 100 = 500$

$\rightarrow 5.05 \cdot 90 = 454.50$

$\Rightarrow$ better to drop the price

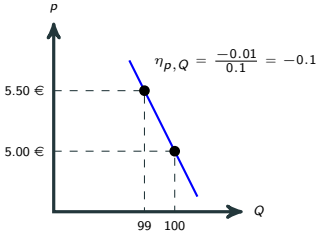


isoelastic

$R(p) = 5 \cdot 100 = 500$

$\rightarrow 5.5 \cdot 90 = 495$

$\Rightarrow$ leave price the same



inelastic

$R(p) = 5 \cdot 100 = 500$

$\rightarrow 5.5 \cdot 99 = 544.50$

$\Rightarrow$ raise the price

Maximising revenue

Suppose a supplier has the market power to manipulate the price, and they have no marginal costs. How would they maximise the **revenue** $R(p) = p \cdot Q(p)$?

We can also demonstrate this mathematically by maximising $R(p)$:

$$0 = \frac{dR}{dp} = \frac{d(p \cdot Q(p))}{dp} = p \cdot \frac{dQ(p)}{dp} + Q(p) = Q\left(\frac{p}{Q}\frac{dQ(p)}{dp} + 1\right) = Q(\eta_{p,Q} + 1)$$

So the revenue is maximised when we reach the isoelastic point $\eta_{p,Q} = -1$.

Electricity demand: typically inelastic

Demand for electricity is largely inelastic.

General reasons for inelasticity:

- Consumers do not perceive the price changes (hidden in monthly bills)
- Lack of substitutes (e.g. for heating: gas, for lighting: gas lamps, for communication: pigeon?, for computation: abacus?)
- Switching to alternative products (substitutes) is cumbersome

Distinguish between **short-run elasticity** and **long-run elasticity**. Long-run elasticity allows consumers time (e.g. years) to become more aware of and purchase alternatives. In the long-run demand tends to be more elastic than the short-run.

E.g. if petrol prices remain high over many years, consumers may be more likely to switch to electric vehicles.

Short-run versus long-run price elasticity

Demand responds more strongly when consumers have more time to adjust to a sustained price change.

Example: road fuel demand

	Short run Usually within 1 year	Long run Adjustment over roughly 5–10 years
Price elasticity η	-0.25	-0.64
Quantity response to a 10% price rise	approx. -2.5%	approx. -6.4%

Short run: drive less or drive more efficiently.

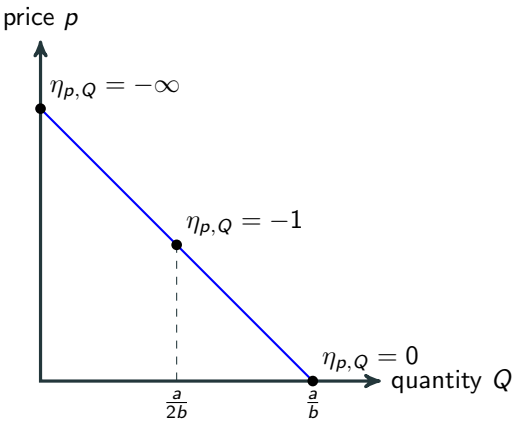
Long run as well: switch to more fuel-efficient vehicles.

Literature-review averages; the time horizons are approximate.

Source: Goodwin et al. (2004), Table 3

Beware: slope is not the same as elasticity

The slope of the inverse demand curve is **not the same** as the elasticity.



For a linear inverse demand function:

$$p(Q) = a - bQ$$

for $a, b > 0$ we have the demand function:

$$Q(p) = \frac{a - p}{b}$$

The elasticity varies with Q:

$$\eta_{p,Q} = \frac{p}{Q}\frac{dQ}{dp} = 1 - \frac{a}{bQ}$$

Positive price-demand elasticity

For some goods, the price-demand elasticity can become **positive**, i.e. rising prices lead to rising demand.

- **Veblen or Snob effect**: A product becomes more attractive the more expensive it is (e.g. exclusive clubs in London, whisky, cigars, Renoirs).
- **Quality effect**: If quality is hard to assess, price is used as a quality indicator (e.g. you want to buy a nice wine for a friend, but don't know enough about it, so choose an expensive one).

Cross elasticity

If the price of one good has an effect on the demand of another good, this indirect elasticity is called a **cross elasticity**. For example, if the price of one good p_1 influences the sales of another Q_2 then the cross elasticity is given by:

$$\eta_{p_1,Q_2} = \frac{p_1}{Q_2}\frac{dQ_2}{dp_1}$$

- Example of **negative** cross-price-elasticity: rising petrol prices leave to sinking demand for internal combustion engine (ICE) cars. Petrol and ICE cars are **complementary goods**.
- Example of **positive** cross-price-elasticity: rising butter prices lead to rising demand for margarine. Butter and margarine are **substitute goods**.

Elasticity: simple example

The price for electricity is 0.2€/kWh. The demand function of a private household (per month) is given by:

$$Q_D(p) = 625 - 625p$$

where the units of Q_D are kWh and of p are €/kWh.

a) How much electricity does the single household consume per month? How much does it pay?

$$Q_D(p = 0.2) = 625 - 0.2 * 625 = 500$$

Consumption: 500 kWh.

$$p \cdot Q_D = 0.2 * 500 = 100$$

Pay per month: 100 €/m.

Elasticity: simple example

The price for electricity is 0.2€/kWh. The demand function of a private household (per month) is given by:

$$Q_D(p) = 625 - 625p$$

where the units of Q_D are kWh and of p are €/kWh.

b) What is the price elasticity at this point? Is it elastic or inelastic?

$$\begin{aligned}\eta_{p,Q} &= \frac{dQ}{dp} \cdot \frac{p}{Q} \\ &= -625 \cdot \frac{0.2}{500} \\ &= -0.25\end{aligned}$$

The demand is inelastic.

Elasticity: simple example

The price for electricity is 0.2€/kWh. The demand function of a private household (per month) is given by:

$$Q_D(p) = 625 - 625p$$

where the units of Q_D are kWh and of p are €/kWh.

c) How does the household react if the price doubles? What are the demand and elasticity now?

$$Q_D(p = 0.4) = 625 - 0.4 * 625 = 375$$

$$\begin{aligned} \eta_{p,Q} &= \frac{dQ}{dp} \cdot \frac{p}{Q} \\ &= -625 \cdot \frac{0.4}{375} \\ &= -0.67 \end{aligned}$$

The demand is still inelastic, but a little more elastic i.e. a little more price-responsive.

Elasticity: simple example

The price for electricity is 0.2€/kWh. The demand function of a private household (per month) is given by:

$$Q_D(p) = 625 - 625p$$

where the units of Q_D are kWh and of p are €/kWh.

d) At what price does the demand become isoelastic?

Solve

$$\begin{aligned} -1 &= \eta_{p,Q} \\ &= \frac{dQ}{dp} \cdot \frac{p}{Q} \\ &= -625 \cdot \frac{p}{625 - 625p} \end{aligned}$$

We find $p = 0.5$ €/kWh and

$$Q_D(p = 0.5) = 625 - 0.5 * 625 = 312.5$$

Production decisions in different types of market

Production decisions are made in the short run

Production is concerned with **how much we produce**, Q, assuming that the capacities of all our factories, machinery and plant are **already determined**.

This is a **short run** decision that we can make in the short term.

In the **long run** we can also change the capacities of production - this is the topic of **investment**, which we will cover later.

Fixed versus variable costs

Fixed costs are costs incurred regardless of the production level Q (e.g. building rent, equipment depreciation); **variable costs** depend on Q and vanish when $Q = 0$ (e.g. energy, materials).

product	fixed costs	variable costs
book	author's effort	paper, glue
e-book	author's effort	almost zero
coal plant	loan repayment, wages	coal
wind plant	loan repayment, wages	almost zero
gas pipeline	loan repayment, wages	fuel for compressor
LNG ship	loan repayment, wages	energy for liquefaction, fuel for ship

Terms and definitions of cost accounting

Consider some production process.

- **Fixed costs** C_{fix} are the share of total costs that do not change when the produced quantity Q is varied, $\frac{dC_{fix}}{dQ} = 0$.
- **Variable costs** $C_{var}(Q)$ are the share of total costs that do change when the produced quantity Q is varied.
- **Total costs** are the sum of fixed and variable costs, $C(Q) = C_{fix} + C_{var}(Q)$.
- **Average costs** are the total costs per unit: $AC(Q) = \frac{C(Q)}{Q}$.
- **Marginal costs** are the costs incurred per unit for an additional unit of production; they depend on current production rate $MC(Q) = \frac{dC}{dQ}$.
- **Revenue/turnover** $p \cdot Q$ is the income from selling at price p.
- **Contribution margin** is the revenue minus variable costs $CM(Q) = p \cdot Q - C_{var}(Q)$, i.e. the contribution towards covering the fixed costs.

Total cost consideration

Consider a firm with 3 product lines A, B, C. Which should they discontinue?

	A	B	C
Turnover	800	500	700
Variable costs	350	150	400
Fixed costs	150	150	500
Total costs	500	300	900
Operating profit	300	200	−200
Total profit		**300**	

Total cost consideration without C

Discontinuing C leads to a worse outcome because fixed costs still need to be paid.

	A	B	C
Turnover	800	500	0
Variable costs	350	150	0
Fixed costs	150	150	500
Total costs	500	300	500
Operating profit	300	200	−500
Total profit		**0**	

Contribution margin

Consider **contribution margin**: revenue/turnover minus variable costs. This is the contribution to covering the fixed costs.

	A	B	C
Turnover	800	500	700
Variable costs	350	150	400
Contribution margin	450	350	**300**
Total contribution margin		1100	
Fixed costs		800	
Total profit		**300**	

Contribution margin: lessons

In the **short-run** it's better to partly cover fixed costs instead of stopping production.

$\Rightarrow$ Ignore fixed costs in the short-run.

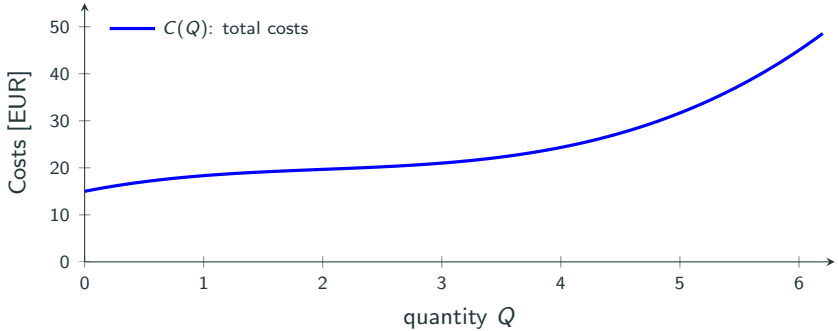
$\Rightarrow$ Maintain production as long as contribution margin (revenue minus variable costs) is positive.

Variable costs can change in the short term (gas, electricity, oil, iron ore, etc.).

In the **long-run** if the contribution margin is persistently less than fixed costs, consider scrapping plant to remove fixed costs (if investment is already paid off).

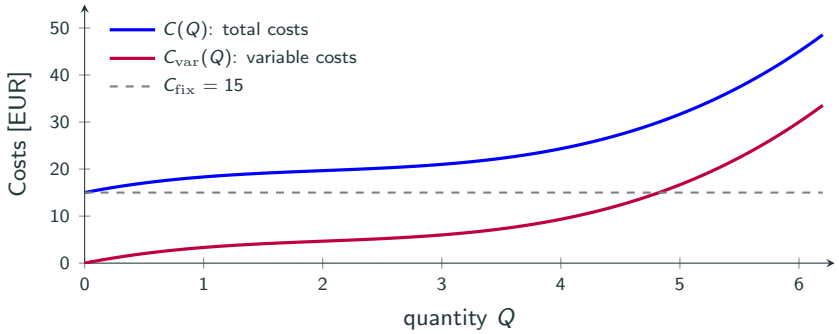
Cost structure: cubic example

Consider a cubic total cost function $C(Q) = \frac{1}{3}Q^3 - 2Q^2 + 5Q + 15$, units EUR.



Cubic example: fixed and variable costs

The **fixed costs** do not depend on Q, $C_{fix} = 15$, while the **variable costs** are the Q-dependent part $C_{var}(Q) = \frac{1}{3}Q^3 - 2Q^2 + 5Q$ with $C(Q) = C_{fix} + C_{var}(Q)$.

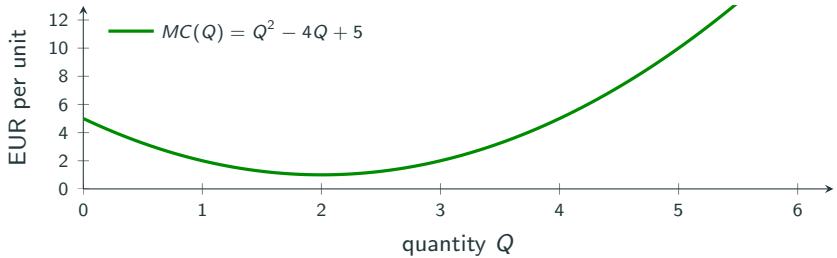


Marginal cost

The **marginal cost** is the cost of the next incremental unit of production. This is the definition of the derivative. Since C_{fix} does not depend on Q, the marginal cost only depends on the variable costs:

$$MC(Q) = \frac{dC(Q)}{dQ} = \frac{dC_{var}(Q)}{dQ}$$

For our cubic example $C(Q) = \frac{1}{3}Q^3 - 2Q^2 + 5Q + 15$, the marginal costs are $\frac{dC_{\text{var}}}{dQ} = Q^2 - 4Q + 5$.



Recall different market forms

Perfect competition:

- many buyers and sellers, none of whom can influence price
- goods homogeneous
- all have perfect information
- no entry or exit barriers

Monopoly:

- single seller who can use **market power** to influence the price

Cournot oligopoly:

- several sellers
- no cooperation/collusion among them

Profit maximisation

A firm wants to maximise its profits Π, i.e. revenue $p \cdot Q$ minus costs $C(Q)$, by changing its production Q:

$$\Pi = p \cdot Q - C(Q)$$

Looking for the maximum $\max_{Q \geq 0} \Pi(Q)$ we find:

$$0 = \frac{d\Pi}{dQ} = \frac{d\,(p \cdot Q)}{dQ} - \frac{dC(Q)}{dQ} = \frac{dp}{dQ} \cdot Q + p - \frac{dC(Q)}{dQ}$$

For **perfect competition** the firm is a **price taker** so that it has no influence on the price $\frac{dp}{dQ} = 0$ and thus:

$$p = \frac{dC(Q)}{dQ}$$

For a given price p from the market, it adjusts its output until its **marginal cost** $\frac{dC(Q)}{dQ}$ is equal to the price. This is a central result of microeconomics!

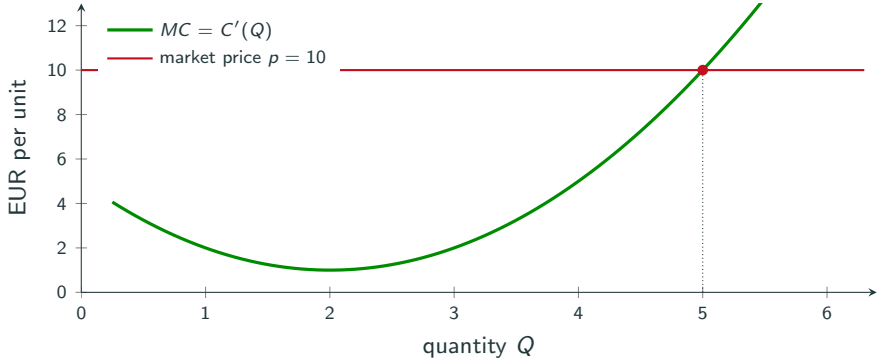
For a **monopoly** where the firm has market power we have to do more work since $\frac{dp}{dQ} \neq 0$.

Profit maximisation under perfect competition: cubic example

Total costs are $C(Q) = \frac{1}{3}Q^3 - 2Q^2 + 5Q + 15$. The producer is a price taker under perfect competition. The market price is 10. What should the production be? Solve:

$$10 = p = \frac{dC_{\text{var}}}{dQ} \Rightarrow 10 = Q^2 - 4Q + 5 \Rightarrow (Q-2)^2 = 9 \Rightarrow Q^* = 2 \pm 3$$

The competitive market price $p = 10$ meets marginal cost on its rising branch at $Q^* = 5$ (negative production is impossible).



Production threshold and profit break-evens

From what price is production worthwhile? And from what price do we make a profit?

The **threshold for production** is the minimum price p_{prod} at which revenue $p \cdot Q$ covers variable costs $C_{\text{var}}(Q)$. At this threshold the contribution margin is zero and we cannot cover any fixed costs. Below this threshold the production should be stopped.

To find the production threshold, find the minimum of the average variable cost:

$$p_{\text{prod}} = \min_{Q>0} \frac{C_{\text{var}}}{Q}$$

Solve:

$$0 = \frac{d\left(\frac{C_{\text{var}}}{Q}\right)}{dQ} = \frac{1}{Q}\frac{dC_{\text{var}}}{dQ} - \frac{1}{Q^2}C_{\text{var}} \Rightarrow \frac{C_{\text{var}}}{Q} = \frac{dC_{\text{var}}}{dQ}$$

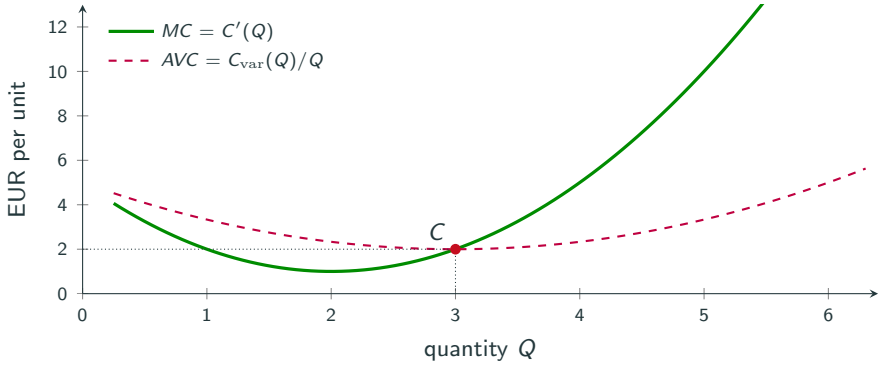
i.e. production threshold is when average variable cost is equal to marginal cost.

Production threshold: cubic example

Production threshold: solve to find production Q_{prod} at threshold:

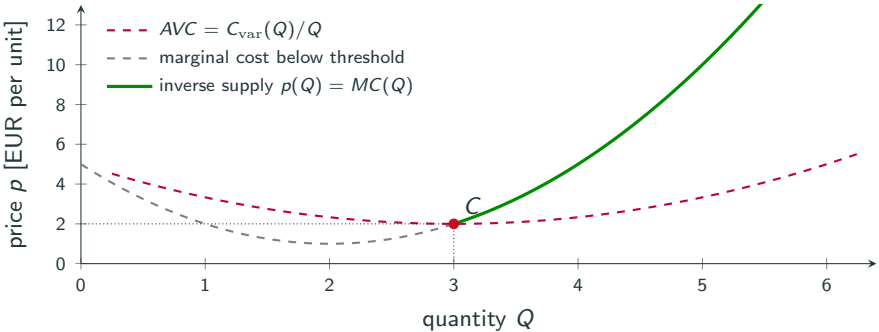
$$\frac{C_{\text{var}}}{Q} = \frac{dC_{\text{var}}}{dQ} \Rightarrow \frac{1}{3}Q^2 - 2Q + 5 = Q^2 - 4Q + 5 \Rightarrow Q_{\text{prod}} = 3$$

Here price is $p_{\text{prod}} = \frac{C_{\text{var}}(Q_{\text{prod}})}{Q_{\text{prod}}} = \frac{dC_{\text{var}}}{dQ} = 2$ at point C.



Relation to the supply curve

The inverse supply function is the marginal cost starting from the production threshold p_{prod}:



At C, $Q_{\text{prod}} = 3$ and $p_{\text{prod}} = 2$. For $p < p_{\text{prod}}$, the firm supplies $Q = 0$.

The **break-even for profit** is the minimum price p_{break} at which revenue $p \cdot Q$ covers total costs $C(Q)$. At this threshold the contribution margin equals the fixed costs.

To find the break-even price, find the minimum of the average total cost:

$$p_{\text{break}} = \min_{Q>0} \frac{C}{Q}$$

Solve:

$$0 = \frac{d\left(\frac{C}{Q}\right)}{dQ} = \frac{1}{Q}\frac{dC}{dQ} - \frac{1}{Q^2}C(Q) \Rightarrow \frac{C}{Q} = \frac{dC_{\text{var}}}{dQ}$$

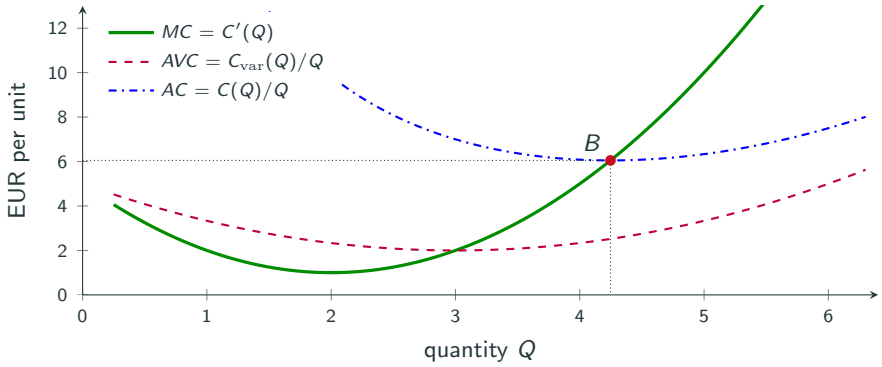
i.e. profit break-even is when average cost is equal to marginal cost.

Profit break-even: cubic example

Break-even: solve to find production Q_{break} at break-even:

$$\frac{C}{Q} = \frac{dC_{\text{var}}}{dQ} \Rightarrow \frac{1}{3}Q^2 - 2Q + 5 + \frac{15}{Q} = Q^2 - 4Q + 5 \Rightarrow Q_{\text{break}} = 4.25$$

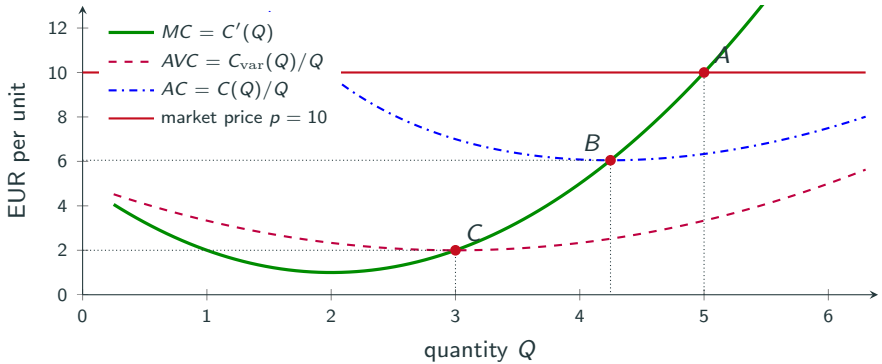
Here price is $p_{\text{break}} = \frac{C(Q_{\text{break}})}{Q_{\text{break}}} = \frac{dC_{\text{var}}}{dQ} \approx 6.05$ at B.



Cost structure: cubic example

If producer is price taker in market with price p, it adjusts its output Q so that $\frac{dC}{dQ} = p$ (A).

Production threshold (C) and profit break-even (B) are also marked.



Monopolies

Profit-maximisation in a monopoly

In a **monopoly** situation there is a single producer that can manipulate the price $p(Q)$ by changing its output Q to maximise its profit. The price is no longer set externally.

Now the profits are given by revenue minus costs:

$$\Pi(Q) = R(Q) - C(Q) = p(Q) \cdot Q - C(Q)$$

Looking for the maximum $\max_{Q \geq 0} \Pi(Q)$ we find:

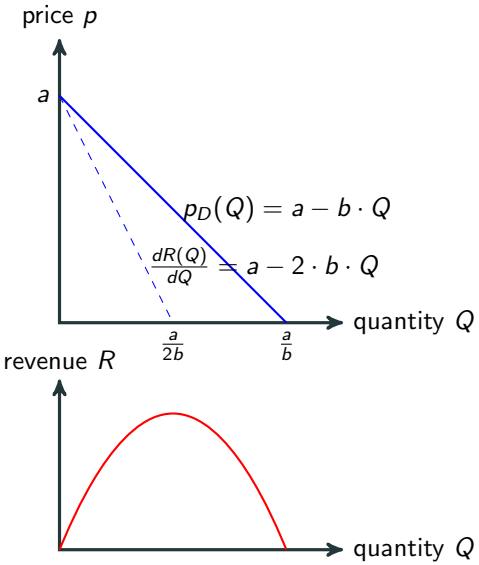
$$0 = \frac{d\Pi}{dQ} = \frac{dR(Q)}{dQ} - \frac{dC(Q)}{dQ} = \frac{d\,(p \cdot Q)}{dQ} - \frac{dC(Q)}{dQ} = \frac{dp}{dQ} \cdot Q + p - \frac{dC(Q)}{dQ}$$

Unlike the case of perfect competition, we now have $\frac{dp}{dQ} \neq 0$ and we find at the optimal point:

$$\frac{dC(Q)}{dQ} = \frac{dR(Q)}{dQ} = p + \frac{dp}{dQ} \cdot Q = p\left(1 + \frac{1}{\eta_{p,Q}}\right)$$

Since the price-demand elasticity is (usually) negative $\eta_{p,Q} < 0$ we can see that in a monopoly that $p > \frac{dC(Q)}{dQ}$, i.e. the price is **higher** than the marginal costs and thus higher than it would be with perfect competition.

Special case: linear demand function



Suppose we are in a market where the consumers can be described by a linear aggregated inverse demand function:

$$p_D(Q) = a - b \cdot Q$$

The revenue $R(Q)$ is given by

$$R(Q) = p_D(Q) \cdot Q = a \cdot Q - b \cdot Q^2$$

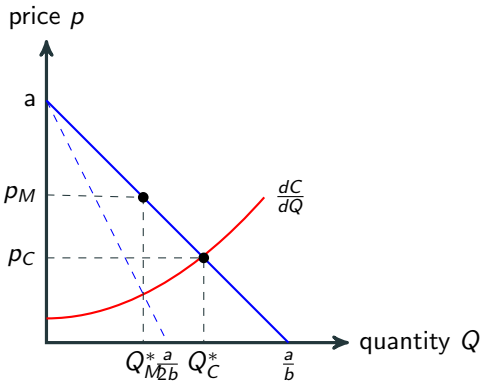
The **marginal revenue** is given by

$$\frac{dR(Q)}{dQ} = a - 2 \cdot b \cdot Q$$

The marginal revenue is linear with double the slope of the inverse demand function.

Monopoly with linear demand function

Consider a monopoly producer with linear inverse demand function $p(Q) = a - b \cdot Q$.



For a monopoly, the optimal production level Q_M^* is set when the marginal revenue equals the marginal cost:

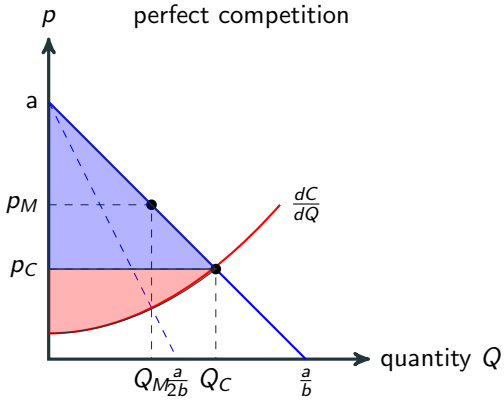
$$\frac{dR(Q)}{dQ} = a - 2 \cdot b \cdot Q = \frac{dC(Q)}{dQ}$$

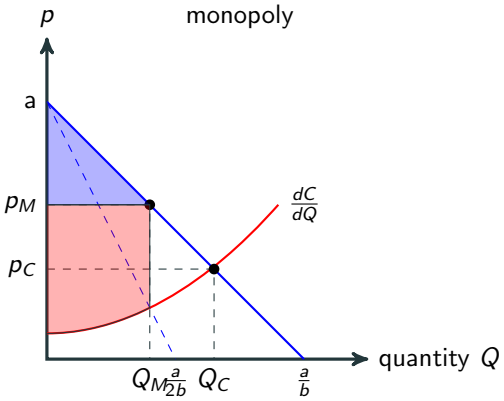
Compare this production level and the resulting price p_M to the case of perfect production where the production level Q_C^* and price p_C are set where the inverse demand function equals the marginal cost:

$$p_D(Q) = a - b \cdot Q = \frac{dC(Q)}{dQ}$$

Welfare in monopoly with linear demand function

Now compare the producer and consumer surpluses in the two cases. In the monopoly situation the producer increases its surplus at the expense of the consumer and the overall total welfare.

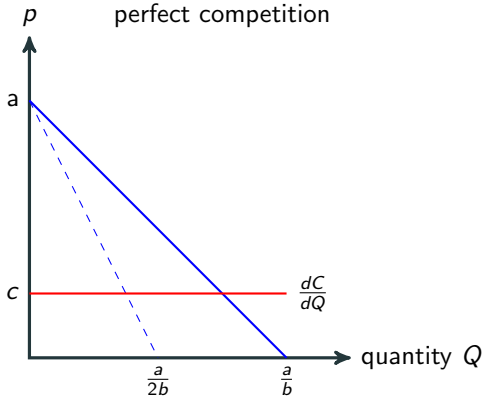


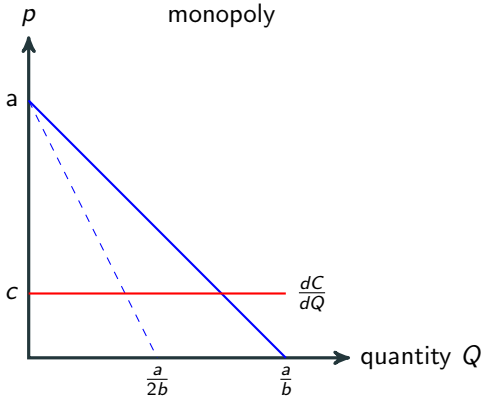


Monopoly example: special case of linear supply costs

Now suppose that as well as a linear demand function $p_D(Q) = a - b \cdot Q$ we also have linear supply costs $C(Q) = c \cdot Q$ so that we have a constant supply function $p_S(Q) = \frac{dC}{dQ} = c$.

What are p_M, p_C, Q_M, Q_C? What about the producer and consumer surpluses?

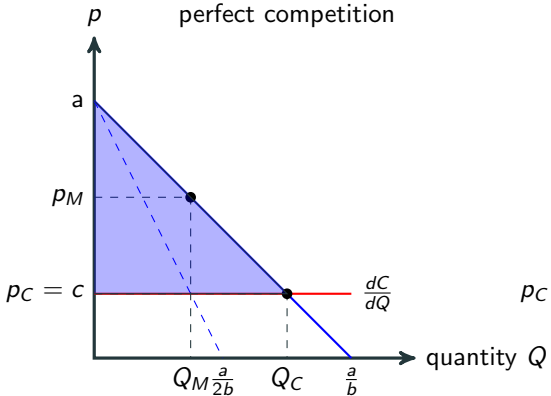


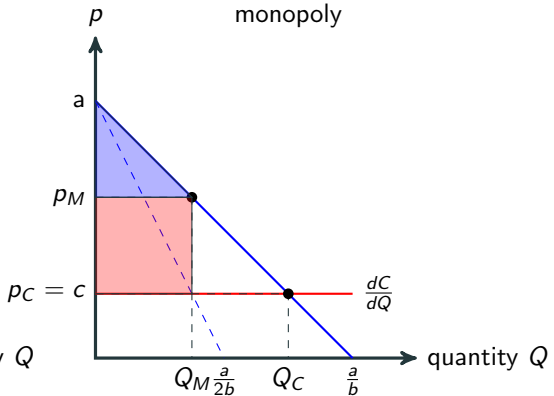


Monopoly example: special case of linear supply costs

For perfect competition $p_C = c$ and $Q_C = \frac{a-c}{b}$. Producer surplus is zero $PS = 0$, while consumer surplus is $CS = \frac{(a-c)^2}{2b}$.

For monopoly $p_M = \frac{a+c}{2}$ and $Q_M = \frac{a-c}{2b}$ (half of perfect competition). Producer surplus is now non-zero $PS = \frac{(a-c)^2}{4b}$ and $CS = \frac{(a-c)^2}{8b}$.





Cournot oligopoly

Cournot oligopoly

A **Cournot oligopoly** is a simplified model that interpolates between the cases of a monopoly (only one supplier) and perfect competition with many suppliers. It has the following properties:

- More than one producing firm
- All firms produce one homogeneous product (no product differentiation)
- No cooperation among firms (no collusion)
- Firms have **market power** - each firm's output decision affects the good's price
- Fixed number of firms
- Firms compete in quantities, and choose them simultaneously
- Economically rational and strategically acting firms, seeking to maximize profit given their competitors' decisions

Cournot duopoly

Two identical firms produce the same good in quantities Q_1, Q_2 with linear supply functions and the same marginal cost:

$$C_1(Q_1) = c \cdot Q_1$$
$$C_2(Q_2) = c \cdot Q_2$$

The price is give through a linear demand function (with $a > c$):

$$p(Q_1, Q_2) = p(Q_1 + Q_2) = a - b \cdot (Q_1 + Q_2)$$

The profit functions of the two firms are given by:

$$\Pi_1(Q_1) = p(Q_1 + Q_2) \cdot Q_1 - C_1(Q_1)$$
$$\Pi_2(Q_2) = p(Q_1 + Q_2) \cdot Q_2 - C_2(Q_2)$$

Cournot duopoly

Consider the maximisation of the profit Π_1 of firm 1 assuming that the production Q_2 of firm 2 is constant and not affected by Q_1:

$$\begin{aligned}\frac{d\Pi_1}{dQ_1} &= p(Q_1+Q_2) + \frac{dp}{dQ_1}\cdot Q_1 - \frac{dC_1}{dQ_1} = a - b\cdot(Q_1+Q_2) - bQ_1 - c \\ &= a - c - 2bQ_1 - bQ_2 = 0\end{aligned}$$

Similarly for the maximisation of the profit Π_2 of firm 2:

$$\begin{aligned}\frac{d\Pi_2}{dQ_2} &= p(Q_1+Q_2) + \frac{dp}{dQ_2}\cdot Q_2 - \frac{dC_2}{dQ_2} = a - b\cdot(Q_1+Q_2) - bQ_2 - c \\ &= a - c - bQ_1 - 2bQ_2 = 0\end{aligned}$$

From the second equation we can substitute $bQ_1 = a - c - 2bQ_2$ into the first to get:

$$Q_2^* = \frac{a-c}{3b} = Q_1^*$$

(note the symmetry between the identical firms) and as a result $p^* = \frac{a+2c}{3}$.

Interpolation from monopoly to polipoly

Using the Cournot model we can interpolate between a monopoly $N = 1$ and perfect competition for the case of linear supply costs $C_i(Q_i) = c \cdot Q_i$ and linear demand function $p_D(\sum_i Q) = a - b \cdot \sum_i Q_i$:

market type	N	price	sales per company	total sales
monopoly	1	$p = \frac{a+c}{2}$	$Q = \frac{a-c}{2b}$	$\frac{1}{2}\frac{a-c}{b}$
duopoly	2	$p = \frac{a+2c}{3}$	$Q_i = \frac{a-c}{3b}$	$\frac{2}{3}\frac{a-c}{b}$
oligopoly	N	$p = \frac{a+Nc}{N+1}$	$Q_i = \frac{a-c}{b(N+1)}$	$\frac{N}{N+1}\frac{a-c}{b}$
polipoly	$N \to \infty$	$p = c$	$\to 0$	$\frac{a-c}{b}$