

Energy Economics, Winter Semester 2025-6

Lecture 10: Resource Management

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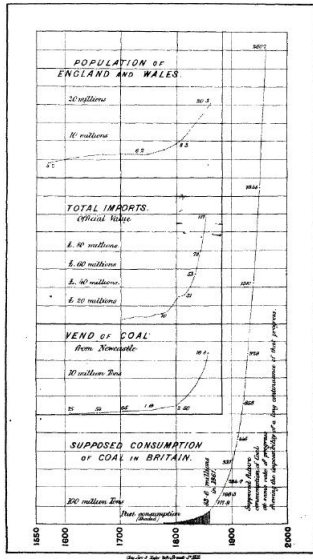
1. Introduction to Resources and Reserves
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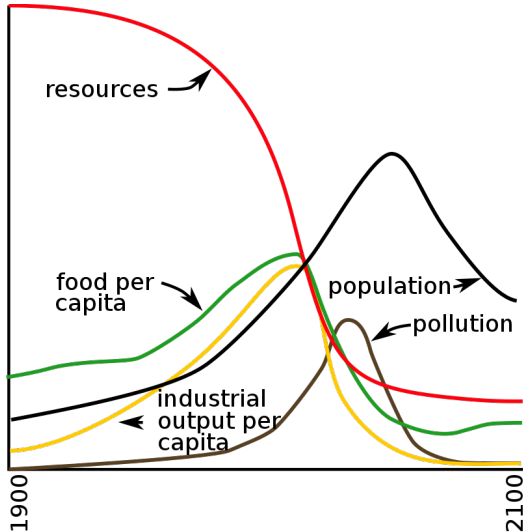
Introduction to Resources and Reserves

Limits to Coal: Jevons' Coal Question in 1865

In 1865 William Stanley Jevons published **The Coal Question**, whose concern was the exhaustion of coal reserves in Britain given exponentially rising demand.

- “I must point out the painful fact that such a rate of growth will before long render our consumption of coal comparable with the total supply. In the increasing depth and difficulty of coal mining we shall meet that vague, but inevitable boundary that will stop our progress.”
- “If we lavishly and boldly push forward in the creation and distribution of our riches, it is hard to over-estimate the pitch of beneficial influence to which we may attain in the present. But the maintenance of such a position is physically impossible. We have to make the momentous **choice between brief greatness and longer continued mediocrity.**”



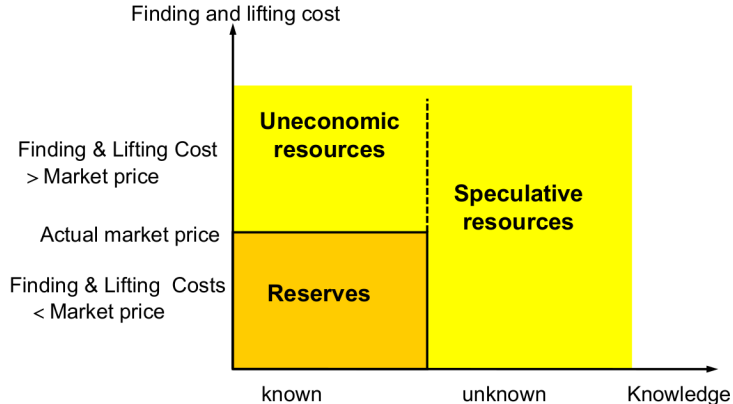


1972 report **The Limits to Growth**, commissioned by the Club of Rome, examined consequences of exponential economic and population growth with a finite supply of resources with a computer simulation.

- Conclusion: “the most probable result will be a rather **sudden and uncontrollable decline** in both population and industrial capacity”.
- But ignores role of technological progress.
- Growth versus limits versus progress: debate continues today.

Resources are all useful raw materials existing in the ground, including those only presumed to exist or now too costly to extract using available technology.

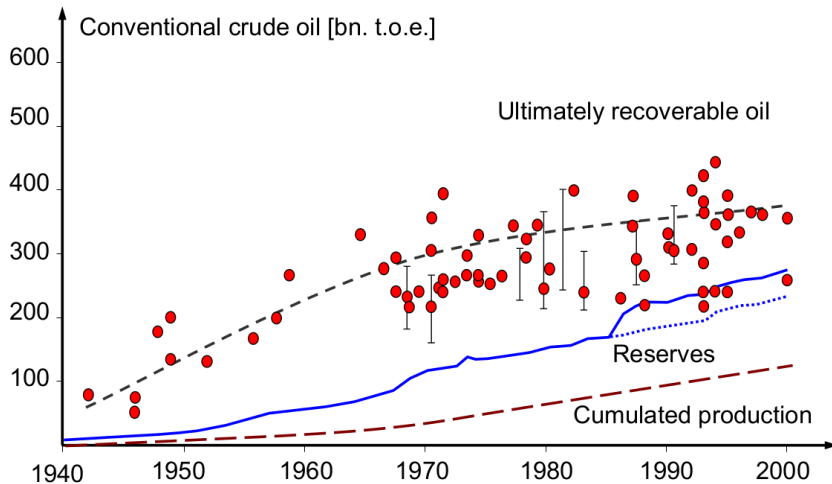
Reserves are those resources known to exist with high probability that can be extracted at a cost below the market price. (\Rightarrow reserves depend on prices and costs.)



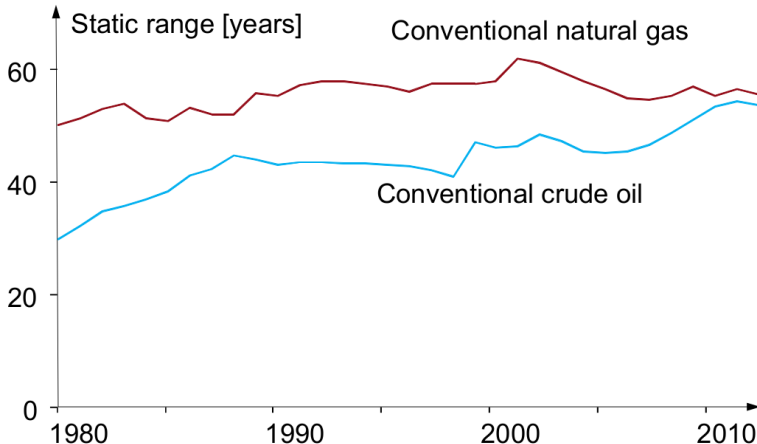
The World Petroleum Council classifies reserves according to the probability of economically viable extraction:

- **Proved (P)**: Probability of extraction $> 90\%$
- **Proved & probable (2P)**: Probability of extraction $> 50\%$
- **Proved & probable & possible (3P)**: Probability of extraction $> 10\%$

Discovery of conventional oil resources over time:



Static range of conventional oil and natural gas reserves (P: Probable > 90%); higher oil prices tend to increase reserves as well as resources by stimulating exploration in the long-term.



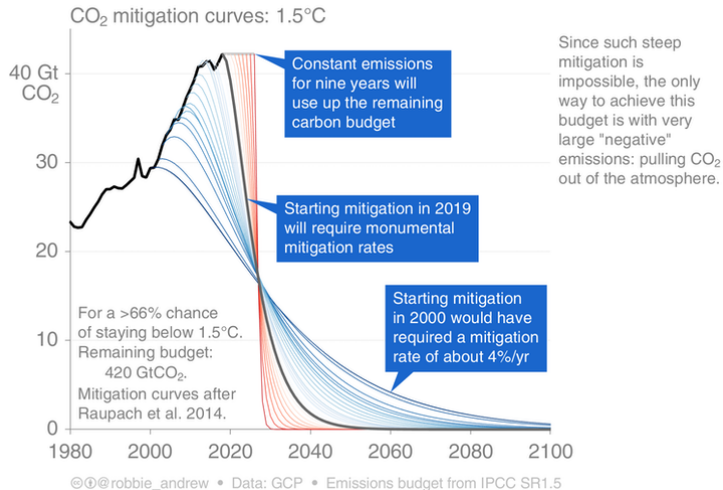
The transformation of reserves into money (i.e. extraction and sale) is an economic decision.

There are two alternatives:

- **wait for higher prices**: leave the reserve in the ground and wait for a higher market price (expected due to increased scarcity)
- **extract resource and invest money**: i.e. put the money in securities or assets thereby earning the market interest rate

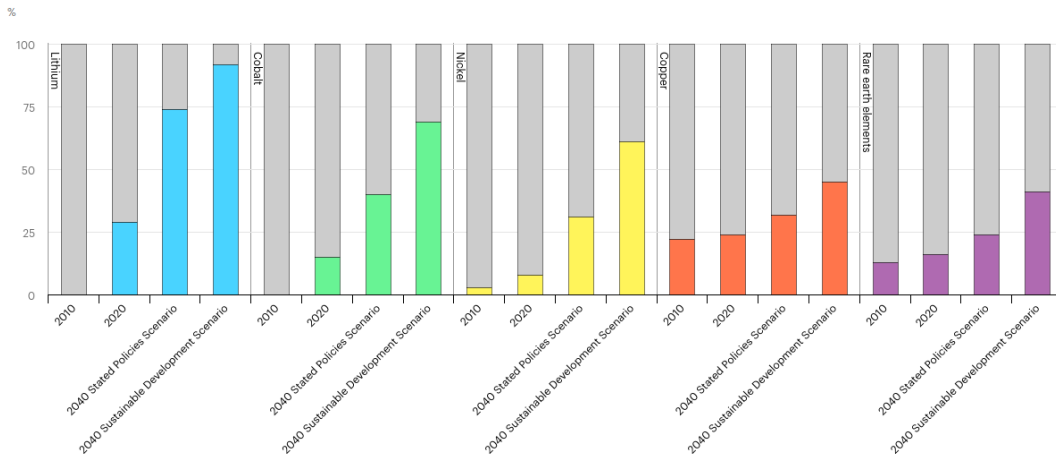
There are opportunity costs to each decision.

Not just hydrocarbons: CO₂ emissions to atmosphere are exhaustible



- We have a carbon budget by 2050 to keep within Paris Agreement.
- E.g. from 2019 there was a budget of 420 GtCO₂ to have at least a 66% chance of staying below 1.5° C.
- How should carbon emissions be managed and priced?
- Should we emit the budget all now or start reducing emissions now to avoid a steeper path later?

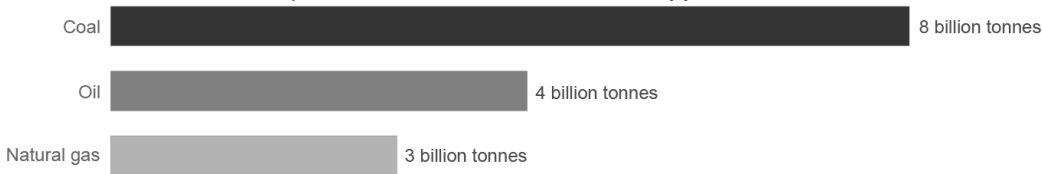
Share of clean energy technologies in total demand for selected minerals by scenario, 2010-2040:



...although the volumes are tiny compared to the fossil world

Fossil fuel production in 2019

The world mines an equivalent of 15 billion tonnes of fossil fuels every year.



Mineral mining for low-carbon energy*

The world mines 7 million tonnes of minerals for all low-carbon tech. In the IEA's Sustainable Development Scenario, which is a rapid deployment of clean energy, this will be 28 million tonnes in 2040.

Production in 2020	7 million tonnes
Projection for 2040	28 million tonnes

*The total mineral production for solar, wind energy, geothermal, hydropower, electric vehicles, battery storage, nuclear, and grid networks.

Data sources: International Energy Agency (IEA); US Energy Information Administration (EIA); BP.

Author: Hannah Ritchie.

Hotelling Rule

Swedish economist Hotelling (1931) developed a simple model to answer the fundamental question for a **finite, non-renewable resource**. Basic assumptions:

- Perfectly Competitive Markets
- Resource Owners:
 - Profit Maximizing Behavior
 - Constant marginal extraction costs c
 - Perfect information about the finite resource stock S

Basic Decision Problem of the Resource Owner:

- The market price p_t cannot be influenced by the resource owner ('price taker'), therefore they just adjust the extraction rate R_t in each period t (e.g. years)
- Profit Π_t in each period follows: $\Pi_t = p_t R_t - c R_t$
- Question: To extract, or not to extract?

- If the profit in the next period, Π_{t+1} , is greater than the profit in the current period times the discount factor $\Pi_t \cdot (1 + i)$, we **do not extract** now but wait

$$\Pi_{t+1} = p_{t+1}R_{t+1} - cR_{t+1} > \Pi_t \cdot (1 + i)$$

- If the profit in the next period, Π_{t+1} , is less than the profit in the current period times the discount factor $\Pi_t \cdot (1 + i)$, we **extract and invest** to earn the interest

$$\Pi_{t+1} = p_{t+1}R_{t+1} - cR_{t+1} < \Pi_t \cdot (1 + i)$$

- If all resource owners behave in a profit-maximising manner, they adjust their extraction rates until:

$$\Pi_{t+1} = p_{t+1}R_{t+1} - cR_{t+1} = \Pi_t \cdot (1 + i)$$

Iterating this idea and assuming that all of our stock S is extracted by the end of the planning period, then the resource owners maximize the Net Present Values of profits by adjusting the extraction rates each period:

$$\max_{R_0, \dots, R_T} NPV = \sum_{t=0}^T \frac{\Pi_t}{(1+i)^t} = \sum_{t=0}^T \frac{(p_t - c)R_t}{(1+i)^t}$$

subject to the available total resource stock S constraint

$$\sum_{t=0}^T R_t = S \tag{1}$$

We can incorporate this constraint in the objective function using a **Lagrange multiplier** λ and maximise the Lagrangian L

$$\max_{R_0, \dots, R_T, \lambda} L = \left[\sum_{t=0}^T \frac{(p_t - c)R_t}{(1+i)^t} - \lambda \left(\sum_{t=0}^T R_t - S \right) \right]$$

The first order optimality conditions satisfied at the optimum are for variable λ

$$\frac{\partial L}{\partial \lambda} = \sum_{t=0}^T R_t - S = 0$$

(just reproducing the constraint), while for each R_t we have

$$\frac{\partial L}{\partial R_t} = \frac{p_t - c}{(1+i)^t} - \lambda = 0$$

This is non-trivial and gives us **Hotelling's Rule**

$$p_t = c + \lambda(1+i)^t$$

In words: the price is always higher than the extraction cost, and increases over time.

Consider **Hotelling's Rule**

$$p_t = c + \lambda(1 + i)^t$$

If the resource is infinite or inexhaustible, $S = \infty$, then the constraint will not be binding and we will have $\lambda = 0$ and a constant price equal to the marginal cost $p_t = c$.

If the resource is finite, then the surcharge over c is called the **scarcity rent** the extractor can claim over the cost:

$$\lambda_t = p_t - c = \lambda(1 + i)^t$$

Note that it grows exponentially!

However, no price can in reality grow forever - there will be some substitute technology.

Eventually p_t will attain the price p_{subst} at which some other energy source becomes competitive as a substitute. This alternative is the **backstop technology**.

This should happen exactly at the end of the period when we have exhausted the supply.

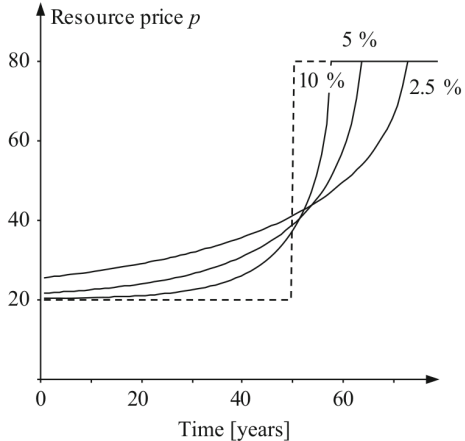
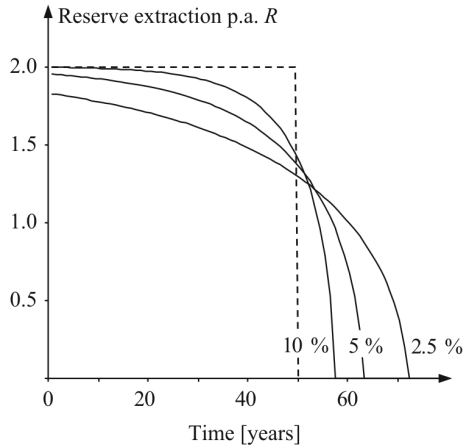
Therefore we assume $p_T = p_{subst}$ from which we deduce

$$p_T = p_{subst} = c + \lambda(1+i)^T \quad \Rightarrow \quad \lambda = \frac{p_{subst} - c}{(1+i)^T}$$

and therefore

$$p_t = c + (p_{subst} - c)(1+i)^{t-T}$$

Suppose we have $c = 20$, $p_{subst} = 80$ and $S = 100$. Price rises faster with higher interest rate i . If demand depends linearly on price, extraction R goes faster with higher i :



Hotelling Rule versus Reality

The EU Emissions Trading System (ETS) puts a cap on how much carbon dioxide can be emitted between now and 2030 by large energy and industrial facilities, and issues certificates for each allowed tCO₂. There is a finite resource of certificates.

We can estimate what price must prevail in 2030 to reach 55% greenhouse gas reduction compared to 1990 levels based on the **marginal abatement cost**. Recent [estimates](#) put it at 130€/tCO₂. If you buy certificates you can either use them now or 'bank' them for later.

We can work back for a given discount rate $i = 0.05$ to find what the price should be now:

$$p_{2022} = \frac{p_{2030}}{(1+i)^8} \sim \frac{130}{1.05^8} = 88$$

This is exactly where the price was at the end of 2021!

Today's CO₂ certificate prices appear to be consistent with the Hotelling rule.

For most extractable resources, the real world does not conform with Hotelling because of the strong simplifying assumptions behind the model.

- New discoveries of reserves and resources increase the stock S over time
- Recycling can reuse already-extracted resources
- Extraction costs c change, either increasing for ever-harder-to-extract sites (e.g. ever deeper coal mines, lower pressure gas/oil), or decreasing due to learning in extraction technology (e.g. fracking for oil and gas)
- Suppliers have a monopoly and can influence price (cf. 1970s oil crises or 2021-2 gas crisis)
- Demand pattern changes (e.g. shift to EVs) or shocks (e.g. Covid) affect price

Oil Price Development Dominated by Demand and Geopolitics



Crude Oil Price History Chart

Price per barrel (Bbl) for West Texas Intermediate (WTI) crude oil; jumps in 2020 and 2022.



Peak Oil/Coal

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- 1919 “...the peak of U.S. production will soon be past – possibly within three years”
 - 1936 “...it is unsafe to rest in the assurance that plenty of petroleum will be found in the future merely because it has been in the past”
 - 1981 “If petroleum is not there to begin with, all of the human ingenuity that can be mustered into the service of exploration cannot put it there. . .”
 - 1990 “...non-OPEC production in the longer term will at best remain stagnant and is more likely to fall gradually due to resource constraints.”
 - 1998 “Global production of conventional oil will begin to decline sooner than most people think probably within 10 years.”
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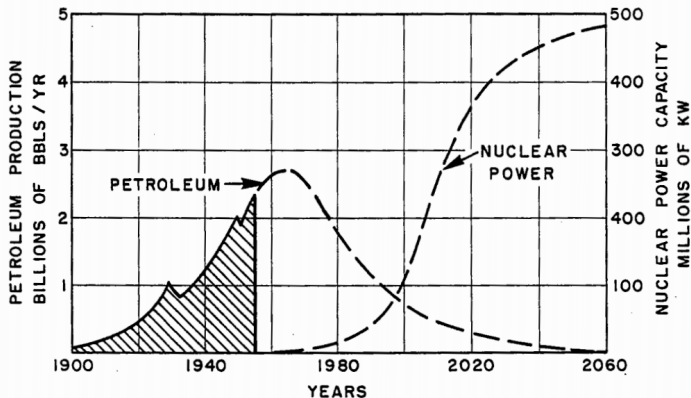


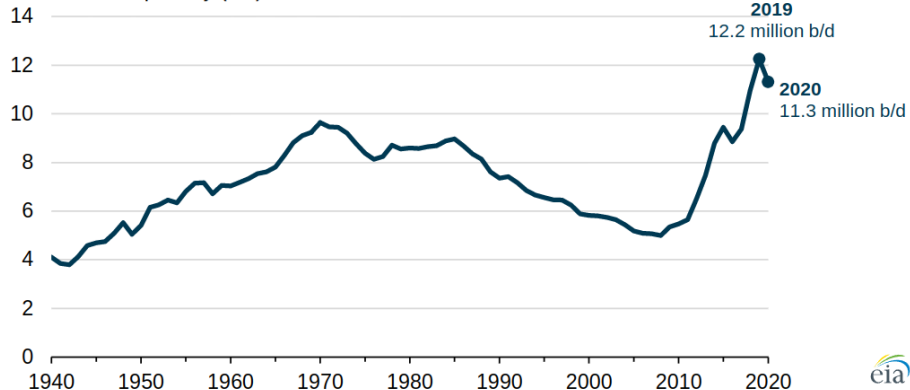
Figure 29 – Concurrent decline of petroleum production and rise of production of nuclear power in the United States. Growth rate of 10 percent per year for nuclear power is assumed; actual rate may be twice this amount.

- Possible scenario projected from 1956 by US geologist M. King Hubbert
- Based on a **logistic diffusion** model, extraction follows a logistic distribution curve (NOT a normal distribution)
- Oil production in the US did indeed peak in the 1970s, but returned to peak height in last decade thanks to shale oil extraction with fracking
- Nuclear stalled

Actual US production peaked in 1970s, but with unconventional sources, such as shale oil and new techniques of horizontal drilling and hydraulic fracturing (fracking), attained new heights in the late 2010s. Need to model basins and resources with separate peaks!

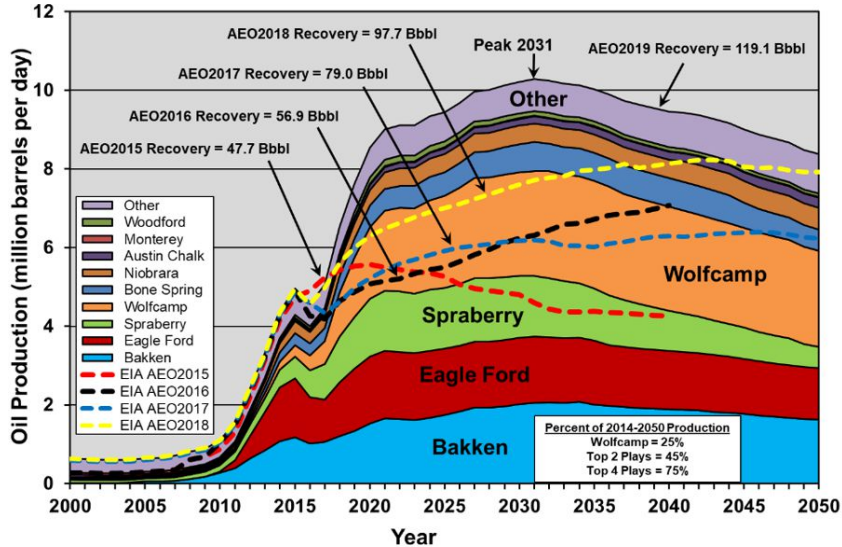
U.S. annual crude oil production (1940–2020)

million barrels per day (b/d)



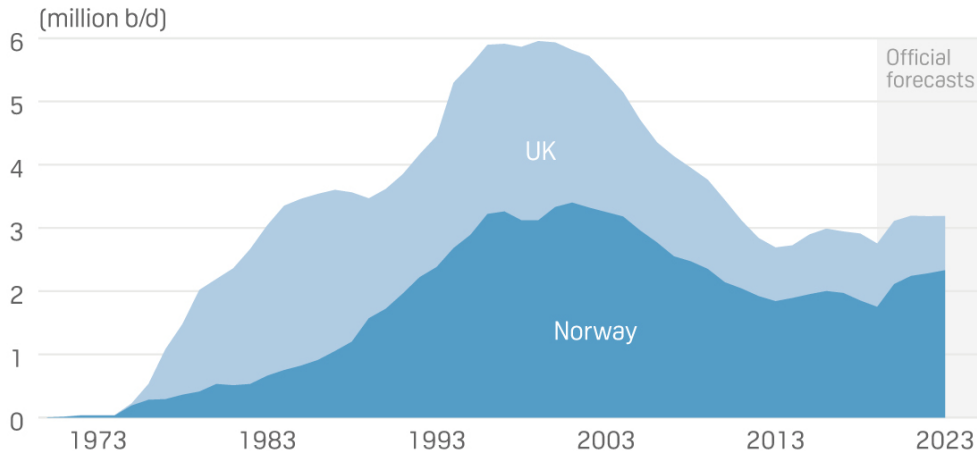
A peak for US shale oil?

Current predictions are that shale oil production could peak in the 2030s.



North Sea oil production (dominated by UK and Norway) appears to have peaked already.

NORTH SEA OIL PRODUCTION ENJOYING LIMITED REVIVAL

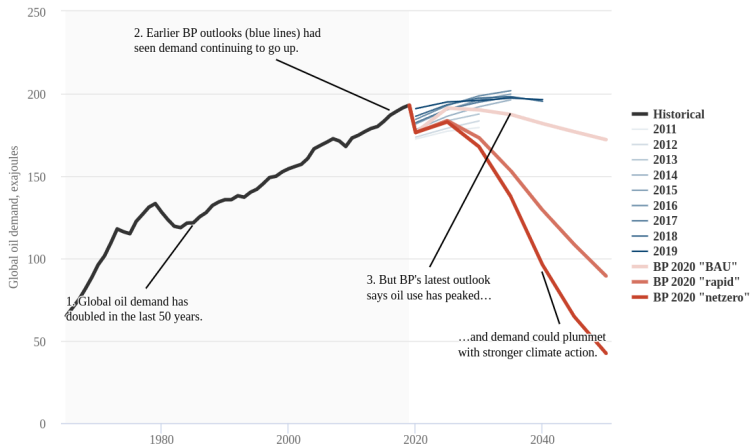


Source: UK's Oil & Gas Authority, Norwegian Petroleum Directorate

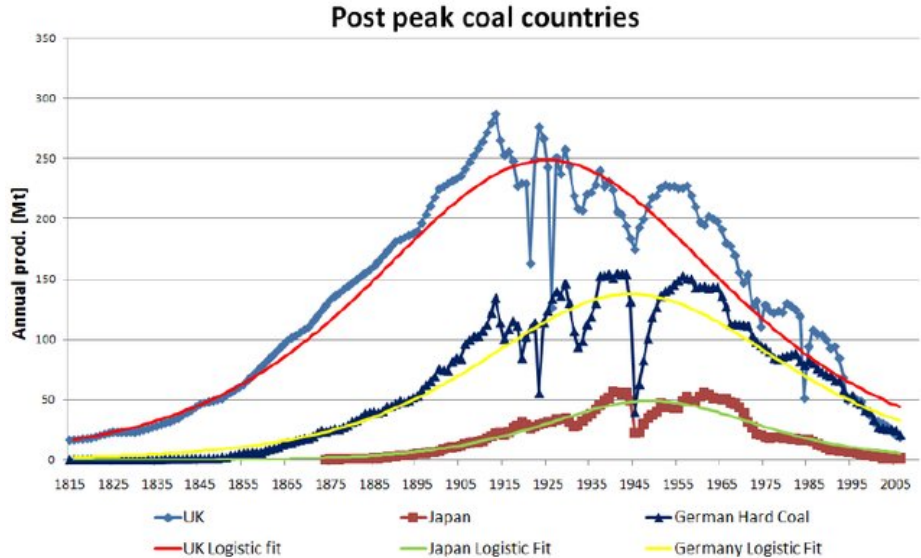
BP suggested in 2020 that worldwide demand may have already peaked. It could sink if electric vehicles eat into demand, but feedstocks for chemical industry will still drive demand.

BP now concedes that oil demand has already peaked – and could soon plummet

Last year's outlook had seen peak oil still being 15 years away



Coal has already peaked in several countries



Logic behind Hubbert curve: total resource is finite, initially production grows exponentially as costs fall with discovery and learning, reach a peak, then decline as marginal costs rise.

For time t , production $P(t)$, total extraction $Q(t)$ and ultimately recoverable resource Q_∞ :

$$\frac{dQ}{dt} = P = \omega Q \left(1 - \frac{Q}{Q_\infty} \right)$$

First equation: total extraction $Q(t)$ is the integral of yearly production.

Second equation: when Q is small, we get $\frac{dQ}{dt} \sim \omega Q$, which gives $Q(t) \sim c \cdot e^{\omega t}$ for some constant c . When Q is large $Q \sim Q_\infty$ we get $\frac{dQ}{dt} = P \sim 0$, i.e. production declines to zero.

Full solution to differential equation is the **logistic distribution curve**

$$P(t) = Q_\infty \omega \frac{1}{\left(e^{-(\omega/2)(\tau-t)} + e^{(\omega/2)(\tau-t)} \right)^2}$$

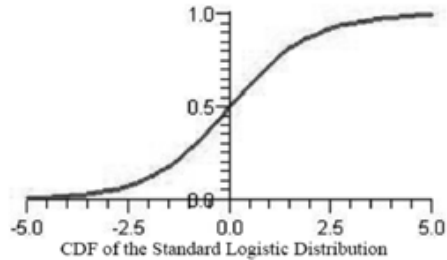
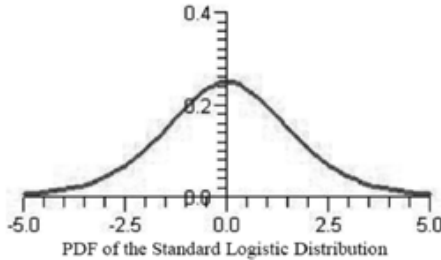
where τ is the time where the production peaks. Hubbert fitted the two parameters Q_∞ and ω to observed production of different resources. NB: decays slower than Gaussian.

The integral of $P(t)$ gives the total extraction $Q(t)$, which is the **logistic curve**

$$Q(t) = \frac{Q_{\infty}}{1 + e^{\omega(\tau-t)}}$$

In the past $t \rightarrow -\infty$ extraction tends to zero $Q(t) \rightarrow 0$. In the future $t \rightarrow \infty$ total extraction approaches the limit $Q(t) \rightarrow Q_{\infty}$.

Left is production $P(t)$, right is cumulated extraction $Q(t)$, for $\tau = 0$:



The view from 5000 years

In his 1956 paper Hubbert imagined the perspective from 5000 years hence, with fossil fuel use a mere blip in our history:

