# Energy Economics, Winter Semester 2023-4
# Lecture 3: Basics of Microeconomics

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# Types of Markets

# Monopoly versus perfect competition

A **market** is where a group of sellers and a group of buyers of a particular good or service come together for exchange.

For **perfect competition** to apply we need:

- **many buyers and many sellers** so nobody can influence the price, i.e. all actors are **price takers**
- goods are exactly the same, i.e. **homogeneous goods**
- all actors have **perfect information**
- **no entry or exit barriers**

In a **monopoly** there is a **single seller**:

- Seller is **sole producer** and can influence the price of its output
- Seller has **market power**: the ability to maintain a price above the price under competition

# Monopoly versus perfect competition

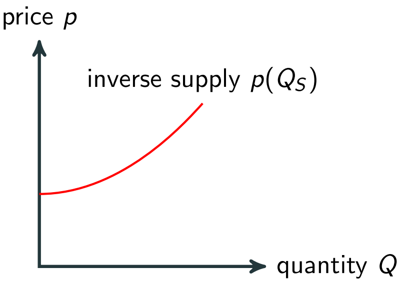
| | one seller | few sellers | many sellers |
|---|---|---|---|
| one buyer | Bilateral monopoly | | Monopsony (buyer's monopoly) |
| few buyers | Oligopolistic market structures | | |
| many buyers | Seller's monopoly | | Perfect competition |

# Perfect Competition

# Inverse supply functions

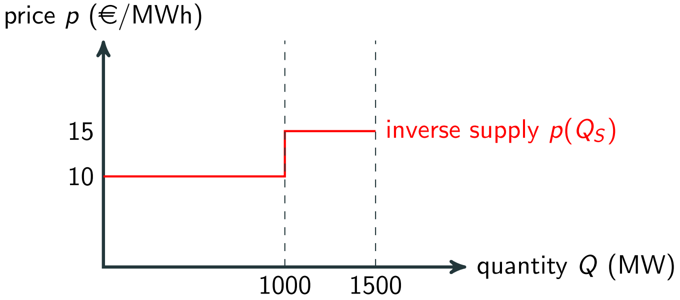
Each producing firm has its own **inverse supply function** which shows at which price $p$ it is worthwhile to sell a given quantity $Q_S$. It indicates the **marginal cost** of the next unit of production at a given production level of $Q_S$ units. Generally: the higher the quantity, the higher the price.

(It's the *inverse* because the supply function is the quantity as a function of price $Q_S(p)$.)



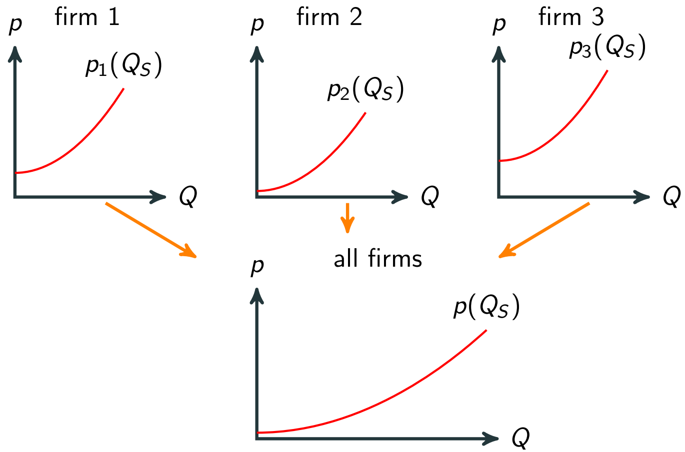
# Inverse supply function: example

A generation firm has two coal plants. The new plant can produce 1000 MW at a cost of 10 €/MWh. An older less efficient plant can produce only 500 MW at a cost of 15 €/MWh.



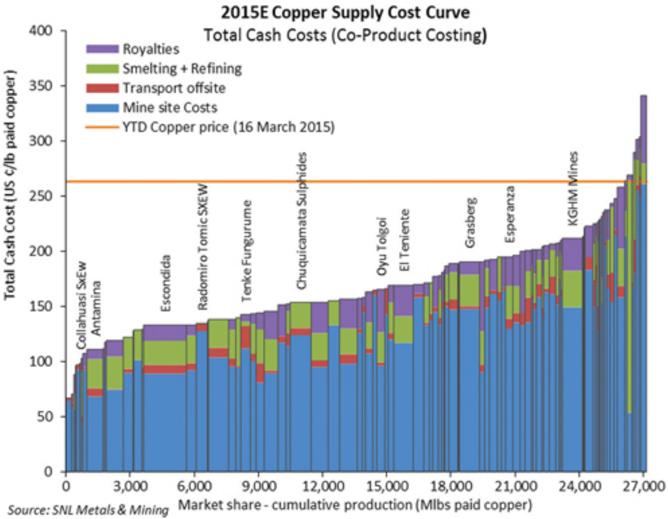
# Aggregation of inverse supply functions

If many firms are active, we can sort and aggregate their inverse supply functions into a single supply function for the whole market.



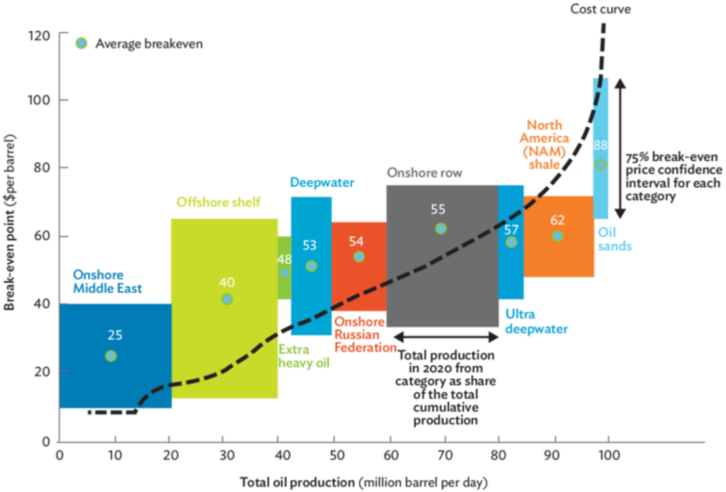
# Example 1: Worldwide aggregated supply curve for copper

Copper is an important component of electrical machinery and distribution.



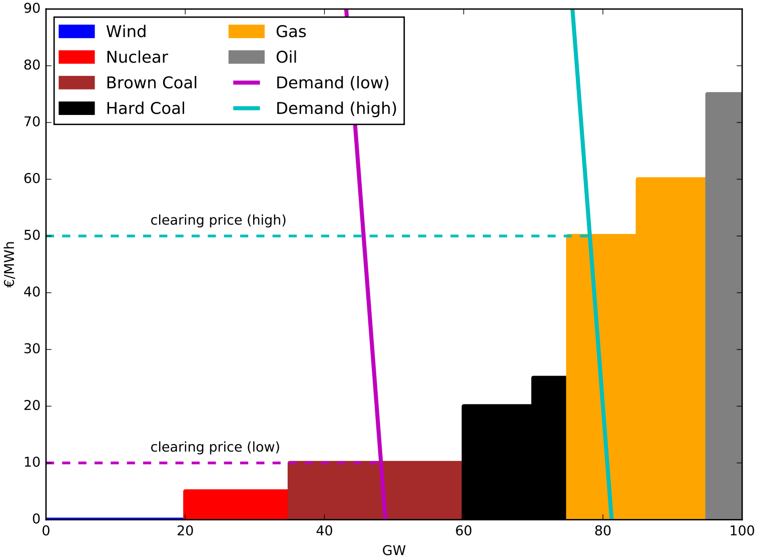
# Example 2: Worldwide aggregated supply function for oil

Boxes indicate ranges for each supply source of oil.




Source: Rystad Energy (2015).

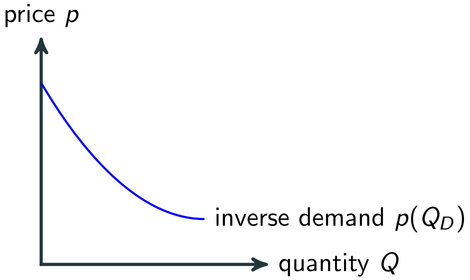
# Example 3: German aggregated supply function for electricity



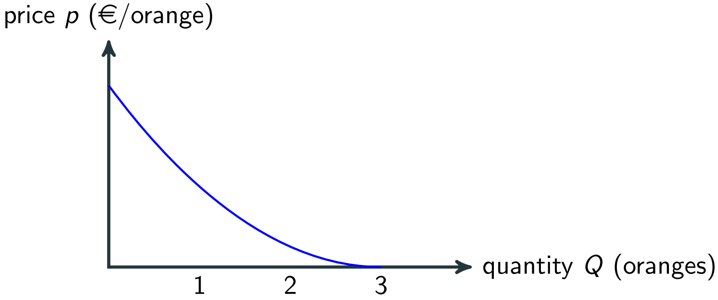
# Inverse demand functions

Each consumer has its own **inverse demand function** which shows which price $p$ it is prepared to pay for a given quantity $Q_D$. It indicates the **willingness to pay** or **marginal utility** of the next unit of consumption at a given consumption level of $Q_D$ units. Generally: the higher the quantity, the lower the price.

(It's the *inverse* because the demand function is the quantity as a function of price $Q_D(p)$.)



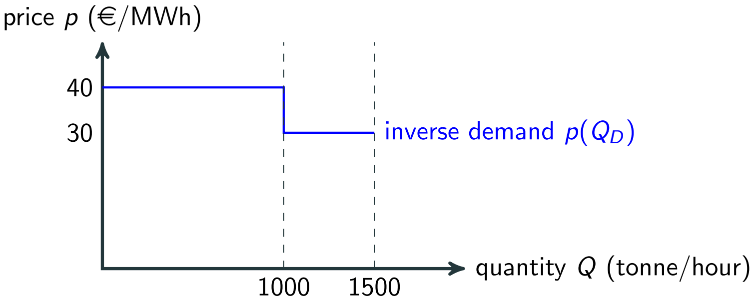
After sport I'm hungry and thirsty. I visit a stand selling oranges. The first orange is worth a lot to me. It partially satisfies my hunger and thirst, so the second is worth less to me. By the 3rd orange, I'm full.



# Inverse demand: example

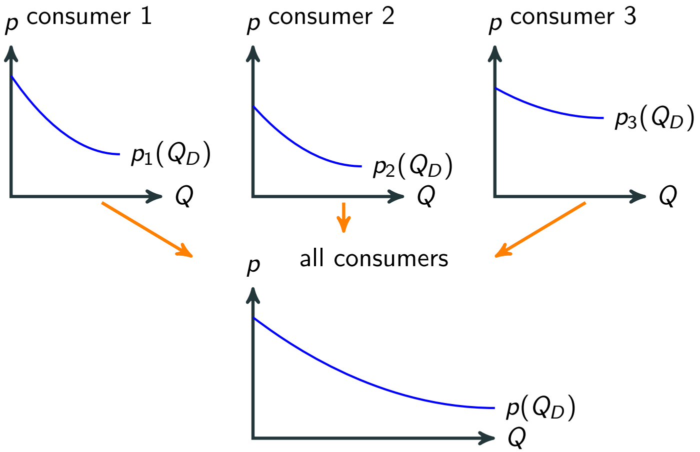
Electrolysis is used to produce aluminium from bauxite ore. On the open market aluminium sells for €1200/tonne. The cost of materials is €600/tonne, which leaves €600/tonne to pay for electricity. A firm has two electrolysis units. A newer efficient one with capacity 1000 tonne/hour consumes 15 MWh/tonne, while an older inefficient one with capacity 500 tonne/hour consumes 20 MWh/tonne. How much are they willing to pay for electricity?

New: $\frac{600€/\mathrm{t}}{15\mathrm{MWh/t}} = 40€/\mathrm{MWh}$, old $\frac{600€/\mathrm{t}}{20\mathrm{MWh/t}} = 30€/\mathrm{MWh}$.



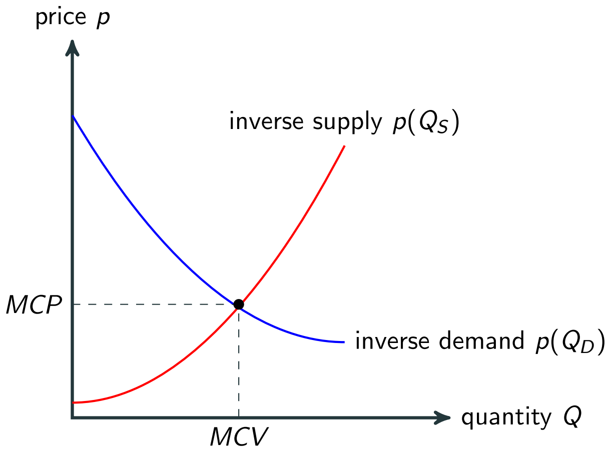
# Aggregation of inverse demand functions

If many consumers are active, we can sort and aggregate their inverse demand functions into a single demand function for the whole market.

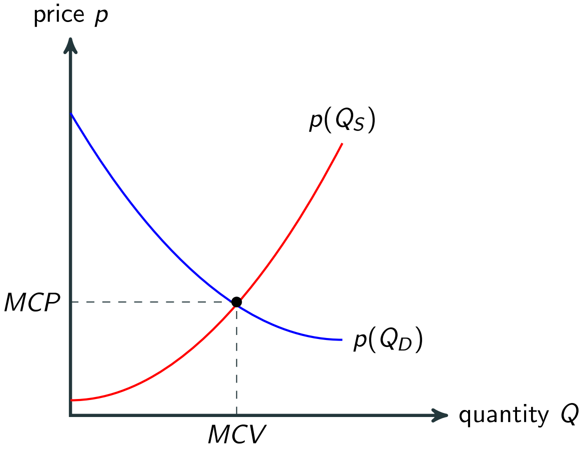


# Market clearing price and volume

In a competitive market the **market clearing price** (MCP) and the **market clearing volume** (MCV) are set by the intersection of the inverse supply and demand functions.



# Market clearing price and volume: important properties




- Every actor sees the same price.
- The price arises **decentrally** from the interaction of supply and demand curves.
- The same amount is supplied as is consumed.
- Since there are many suppliers and consumers, no single actor can influence the price (they are all **price takers**).
- The price is higher than the costs of each supplier whose offer is taken.
- The price is lower than the willingness to pay of each consumer whose bid is taken.

# Market clearing price and volume: simple example

Consider a market with:

Aggregated inverse demand curve $p_D(Q_D) = 28 - 4Q_D$.

Aggregated inverse supply curve $p_S(Q_S) = 1 + 5Q_S$.

Q: What is the market clearing price and volume?

Solve

$$28 - 4Q = 1 + 5Q$$

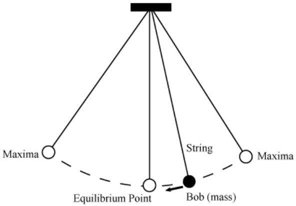
Find $Q^* = 3, p^* = 16$.

# Price finding as an equilibrium

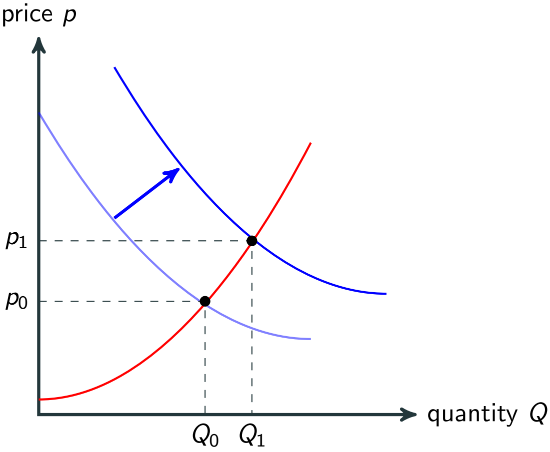
The price finding mechanism is often called an **equilibrium** in analogy to physics because the 'forces' of supply and demand are equal.

In physics, a **stable equilibrium** (like a pendulum) has negative feedback that returns the system to equilibrium after a disturbance. An **instable equilibrium** (like a pencil on its end) would not return to its equilibrium.

A market equilibrium is **stable** because the market actors have no incentive to change their behaviour, since they are already optimally adapted to the relevant market data.

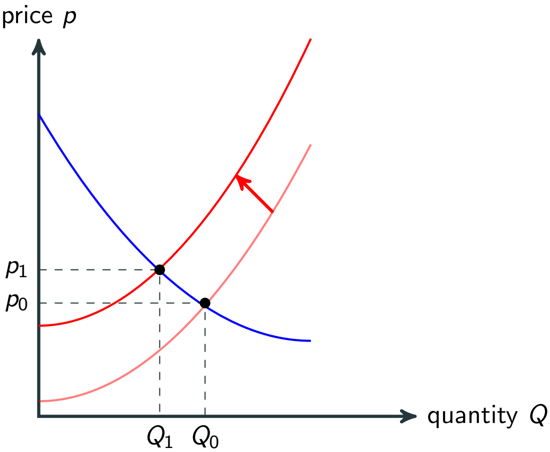



If demand increases in volume and/or willingness to pay, the price goes up.



# Reaction to increase in supply cost

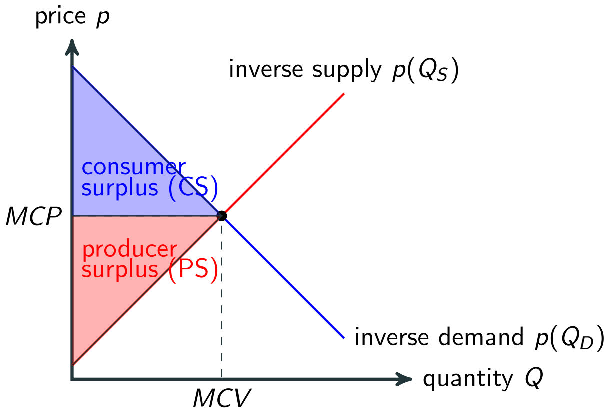
If supply becomes more expensive or decreases in volume, the price goes up.



# Consumer and producer surpluses

The **consumer surplus** is the total amount consumers are willing to pay minus what they actually pay. The **producer surplus** in the total revenue for producers minus their actual costs.

The **total surplus** or **total welfare** $TW = CS + PS$ indicates the degree of efficiency of resource allocation.



# Consumer and producer surpluses: example

Consider the previous example:

Aggregated inverse demand curve $p_D(Q_D) = 28 - 4Q_D$.

Aggregated inverse supply curve $p_S(Q_S) = 1 + 5Q_S$.

MCV and MCP are $Q^* = 3, p^* = 16$.

Q: What are the consumer and producer surpluses?

Consumer surplus: $CS = 0.5 * 3 * (28 - 16) = 18$.

Producer surplus: $PS = 0.5 * 3 * (16 - 1) = 22.5$.

Total welfare: $TW = CS + PS = 18 + 22.5 = 40.5$.

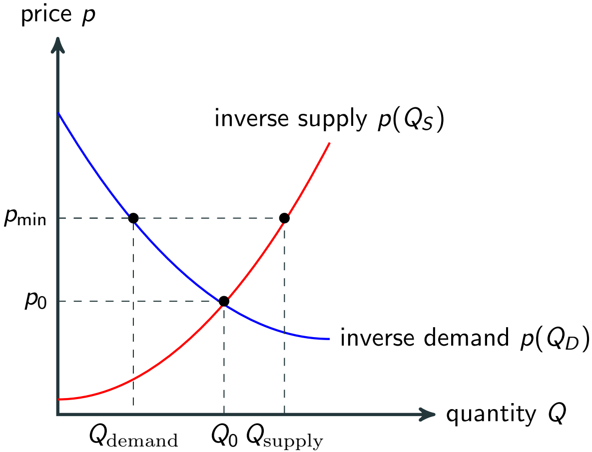
# Market failures

Markets can fail to deliver welfare-maximisation if the conditions for perfect competition are not met. This can justify **state intervention**. Examples of **market failures** include:

- **Monopoly/monopsony**: Single actors dominate and use **market power** to set prices
- **Asymmetric information**: Some actors have more information than others (e.g. insider trading, used car salesman deliberately selling faulty goods)
- **Inhomogeneous goods**: Quality differences (e.g. fake Gucci handbags)
- **Market entry/exit barriers**: Examples include taxi cartels, notaries
- **Externalities ignored**: Costs are induced on third parties not active in the market, which are not priced in (e.g. air pollution, greenhouse gases)
- **Public goods**: Everyone benefits from them, but nobody has an incentive to produce them - **free-rider problem** (e.g. peace, biodiversity, dikes, education)

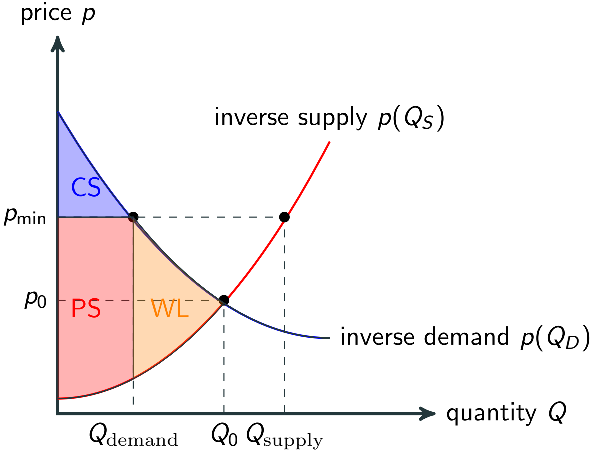
# Reaction to state regulation: minimum price

A **minimum price** or **floor price** can lead to lower demand and/or excess supply (if there is a buyer of last resort like the state). Cf. butter mountains and milk lakes in EU in past; minimum wages.

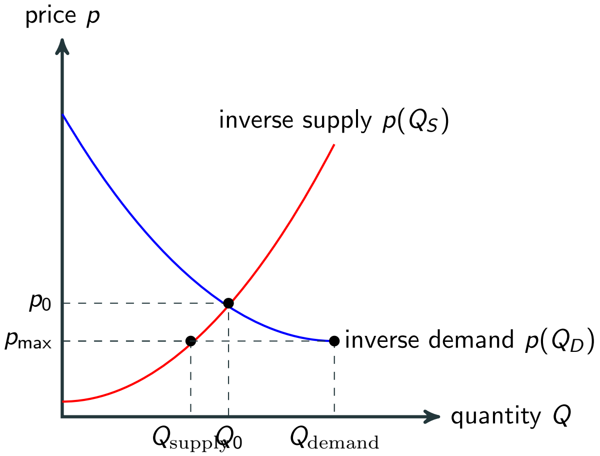


# Minimum price: effect on total welfare

The introduction of a minimum price will **reduce** total welfare in this market, although it may have benefits in other parts of society (e.g. economic security for farmers and a living wage for workers). The **welfare loss (WL)** or **deadweight loss** corresponds to the orange triangle.

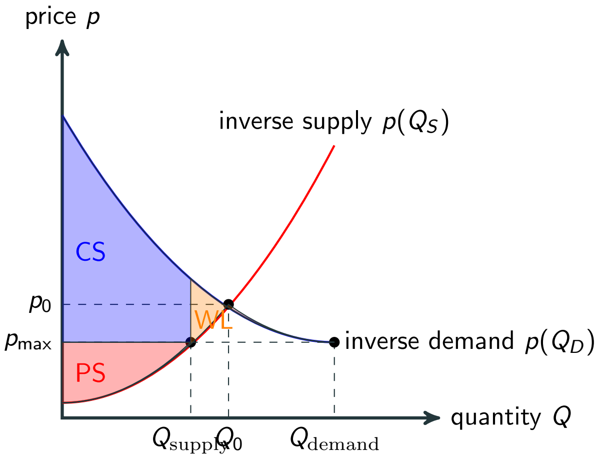


A **maximum price** or **price cap** can lead to lower supply and/or excess demand. Cf. rent caps, energy price caps.



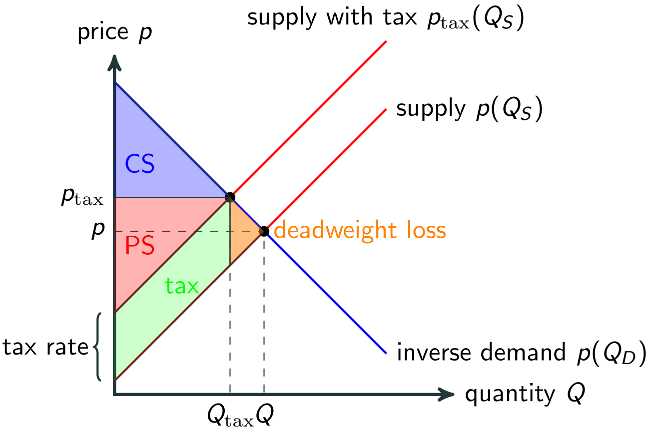
# Maximum price: effect on total welfare

The introduction of a maximum price will **reduce** total welfare in this market, although it may have benefits in other parts of society (e.g. benefits for consumers). The **welfare loss (WL)** or **deadweight loss** corresponds to the orange triangle.



# Consumer and producer surpluses: response to tax

A **tax** leads to **tax revenue** for the state, which counts towards the total welfare. However there is a **welfare loss** or **deadweight loss** compared to the case without the tax.



# Market clearing price and volume with tax: simple example

Consider a market with:

Aggregated inverse demand curve $p_D(Q_D) = 28 - 4Q_D$.

Aggregated inverse supply curve $p_S(Q_S) = 1 + 5Q_S$.

Suppose we tax the product at a rate of 9€/unit. What is the market clearing price and volume now?

Solve

$$28 - 4Q = 10 + 5Q$$

Find $Q^* = 2, p^* = 20$.

# Market clearing price and volume with tax: simple example

Consider a market with:

Aggregated inverse demand curve $p_D(Q_D) = 28 - 4Q_D$.

Aggregated inverse supply curve $p_S(Q_S) = 1 + 5Q_S$.

Suppose we tax the product at a rate of 9€/unit. How do the surpluses change?

Consumer surplus: $CS_{\mathrm{tax}} = 0.5 * 2 * (28 - 20) = 8$.

Producer surplus: $PS_{\mathrm{tax}} = 0.5 * 2 * (20 - 10) = 10$.

Total tax revenue: $TR_{\mathrm{tax}} = 2 * 9 = 18$.

Total welfare includes tax revenue: $TW_{\mathrm{tax}} = CS + PS + TR = 36$.

Deadweight loss: $TW - TW_{\mathrm{tax}} = 40.5 - 18 - 18 = 4.5$.

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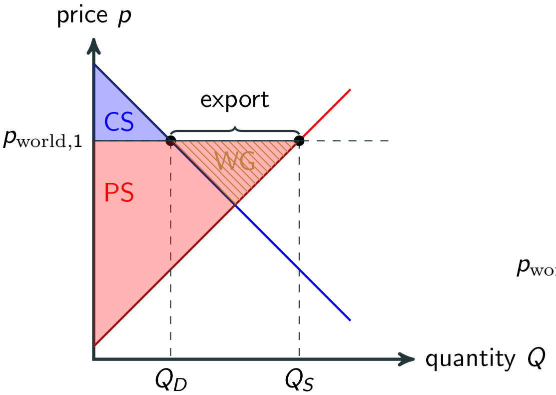
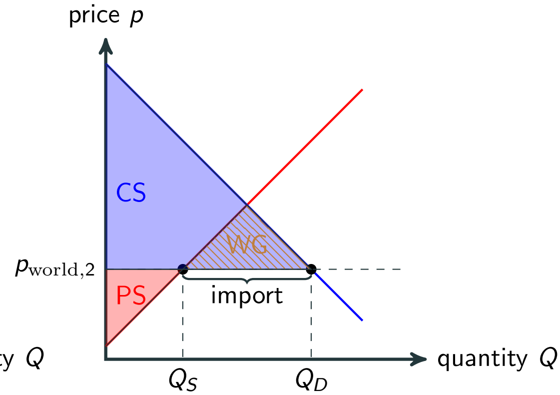
Trade between 2 regions always leads to a **welfare gain** (WG) in each country, but can have a strong influence on the distribution between the consumer and producer surpluses.



# Elasticity

**Elasticity** is a is a measure of how much buyers and sellers respond to changes in market conditions.

**Price elasticity** measures the response to price changes. Price elasticity of demand $\eta_{p,Q}$ is a measure of how much the quantity demanded of a good responds to a change in the price of that good.

$$\text{Price elasticity of demand} = \frac{\text{\% change in quantity demanded}}{\text{\% change in price}}$$

$$\eta_{p,Q} = \frac{\frac{dQ}{Q}}{\frac{dp}{p}} = \frac{dQ}{dp} \cdot \frac{p}{Q}$$

# Elastic versus inelastic demand

Demand-price elasticity is generally negative: the higher the price, the lower the demand. The magnitude of the elasticity allows it to be classified:

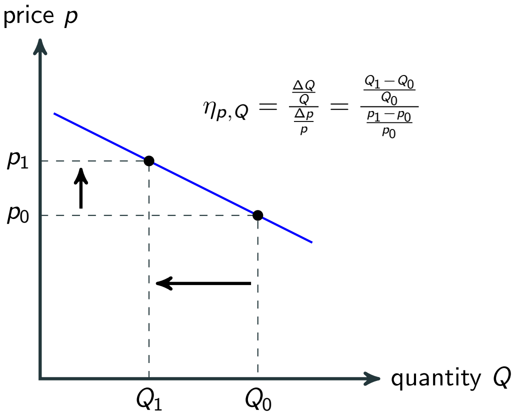
$-\infty < \eta_{p,Q} \leq -1$ **elastic demand** (big change in demand for small change in price)

$\eta_{p,Q} = -1$ **isoelastic demand** (same % change in demand for % change in price)

$-1 < \eta_{p,Q} \leq 0$ **inelastic demand** (small change in demand for large change in price)

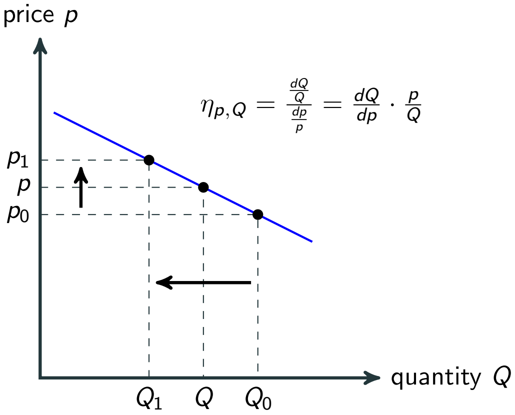
If demand does not respond at all to prices, $\eta_{p,Q} = 0$, it is **perfectly inelastic**.

The **arc elasticity** can be measured if you know how the demand responds to a specific price increase from $p_0$ to $p_1$.



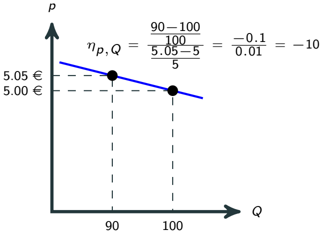
# Point elasticity

The **point elasticity** is the infinitesimal version if you can precisely calculate the derivative.

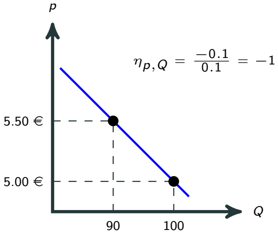


# Examples of price-demand elasticity

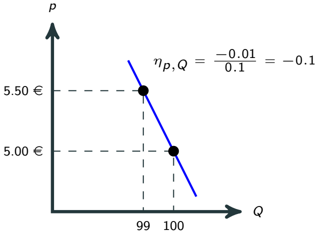
Consider the following arc elasticities:




**very elastic** (small change in price ⇒ big change in demand)

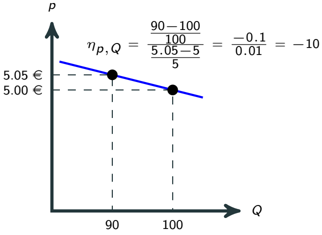


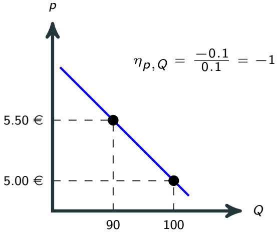

**isoelastic** (change in price ⇒ propotional change in demand)

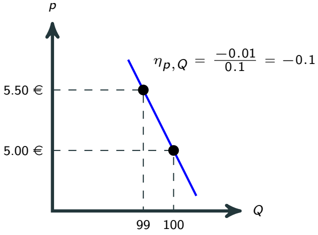



**very inelastic** (large change in price ⇒ small change in demand)

Suppose a supplier has the market power to manipulate the price, and they have no marginal costs. How to maximise the **revenue** $R(p) = p \cdot Q(p)$? Consider effect of price increase:




**elastic**

$R(p) = 5 \cdot 100 = 500$

$\rightarrow 5.05 \cdot 90 = 454.50$

$\Rightarrow$ better to drop the price

**isoelastic**

$R(p) = 5 \cdot 100 = 500$

$\rightarrow 5.5 \cdot 90 = 495$

$\Rightarrow$ leave price the same

**inelastic**

$R(p) = 5 \cdot 100 = 500$

$\rightarrow 5.5 \cdot 99 = 544.50$

$\Rightarrow$ raise the price

# Maximising revenue

Suppose a supplier has the market power to manipulate the price, and they have no marginal costs. How would they maximise the **revenue** $R(p) = p \cdot Q(p)$?

We can also demonstrate this mathematically by maximising $R(p)$:

$$0 = \frac{dR}{dp} = \frac{d(p \cdot Q(p))}{dp} = p \cdot \frac{dQ(p)}{dp} + Q(p) = Q\left(\frac{p}{Q}\frac{dQ(p)}{dp} + 1\right) = Q(\eta_{p,Q} + 1)$$

So the revenue is maximised when we reach the isoelastic point $\eta_{p,Q} = -1$.

# Electricity demand: typically inelastic

Demand for electricity is largely inelastic.

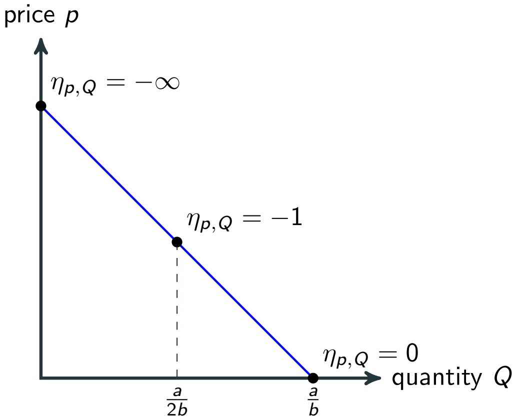
General reasons for inelasticity:

- Consumers do not perceive the price changes (hidden in monthly bills)
- Lack of substitutes (e.g. for heating: gas, for lighting: gas lamps, for communication: pigeon?, for computation: abacus?)
- Switching to alternative products (substitutes) is cumbersome

Distinguish between **short-run elasticity** and **long-run elasticity**. Long-run elasticity allows consumers time (e.g. years) to become more aware of and purchase alternatives. In the long-run demand tends to be more elastic than the short-run.

E.g. if petrol prices remain high over many years, consumers may be more likely to switch to electric vehicles.

The slope of the inverse demand curve is **not the same** as the elasticity.




For a linear inverse demand function:

$$p(Q) = a - bQ$$

for $a, b > 0$ we have the demand function:

$$Q(p) = \frac{a-p}{b}$$

The elasticity varies with $Q$:

$$\eta_{p,Q} = \frac{p}{Q}\frac{dQ}{dp} = 1 - \frac{a}{bQ}$$

# Positive price-demand elasticity

For some goods, the price-demand elasticity can become **positive**, i.e. rising prices lead to rising demand.

- **Veblen or Snob effect**: A product becomes more attractive the more expensive it is (e.g. exclusive clubs in London, whisky, cigars, Renoirs).
- **Quality effect**: If quality is hard to assess, price is used as a quality indicator (e.g. you want to buy a nice wine for a friend, but don't know enough about it, so choose an expensive one).

# Cross elasticity

If the price of one good has an effect on the demand of another good, this indirect elasticity is called a **cross elasticity**. For example, if the price of one good $p_1$ influences the sales of another $Q_2$ then the cross elasticity is given by:

$$\eta_{p_1,Q_2} = \frac{p_1}{Q_2}\frac{dQ_2}{dp_1}$$

- Example of **negative** cross-price-elasticity: rising petrol prices leave to sinking demand for cars. Petrol and cars are **complementary goods**.
- Example of **positive** cross-price-elasticity: rising butter prices lead to rising demand for margarine. Butter and margarine are **substitute goods**.

# Elasticity: simple example

The price for electricity is 0.2€/kWh. The demand function of a private household (per month) is given by:

$$Q_D(p) = 625 - 625p$$

where the units of $Q_D$ are kWh and of $p$ are €/kWh.

a) How much electricity does the single household consume per month? How much does it pay?

$$Q_D(p = 0.2) = 625 - 0.2 * 625 = 500$$

Consumption: 500 kWh.

$$p \cdot Q_D = 0.2 * 500 = 100$$

Pay per month: 100 €/m.

The price for electricity is 0.2€/kWh. The demand function of a private household (per month) is given by:

$$Q_D(p) = 625 - 625p$$

where the units of $Q_D$ are kWh and of $p$ are €/kWh.

b) What is the price elasticity at this point? Is it elastic or inelastic?

$$\begin{aligned}\eta_{p,Q} &= \frac{dQ}{dp} \cdot \frac{p}{Q} \\ &= -625 \cdot \frac{0.2}{500} \\ &= -0.25\end{aligned}$$

The demand is inelastic.

# Elasticity: simple example

The price for electricity is 0.2€/kWh. The demand function of a private household (per month) is given by:

$$Q_D(p) = 625 - 625p$$

where the units of $Q_D$ are kWh and of $p$ are €/kWh.

c) How does the household react if the price doubles? What are the demand and elasticity now?

$$Q_D(p = 0.4) = 625 - 0.4 * 625 = 375$$

$$\begin{aligned}\eta_{p,Q} &= \frac{dQ}{dp} \cdot \frac{p}{Q} \\ &= -625 \cdot \frac{0.4}{375} \\ &= -0.67\end{aligned}$$

The demand is still inelastic, but a little more elastic i.e. a little more price-responsive.

# Elasticity: simple example

The price for electricity is 0.2€/kWh. The demand function of a private household (per month) is given by:

$$Q_D(p) = 625 - 625p$$

where the units of $Q_D$ are kWh and of $p$ are €/kWh.

d) At what price does the demand become isoelastic?

Solve

$$\begin{aligned} -1 &= \eta_{p,Q} \\ &= \frac{dQ}{dp} \cdot \frac{p}{Q} \\ &= -625 \cdot \frac{p}{625 - 625p} \end{aligned}$$

We find $p = 0.5$ €/kWh and

$$Q_D(p = 0.5) = 625 - 0.5 * 625 = 312.5$$

# Production decisions in different types of market

# Production decisions are made in the short run

**Production** is concerned with **how much we produce**, $Q$, assuming that the capacities of all our factories, machinery and plant are **already determined**.

This is a **short run** decision that we can make in the short term.

In the **long run** we can also change the capacities of production - this is the topic of **investment**, which we will cover later.

# Supply side: direct versus opportunity costs

The price at which a seller is willing to sell their goods is determined by their costs of production:

- **Explicit, direct costs**: out-of-pocket expenses, money actually paid e.g. for wages, materials, energy
- **Opportunity costs**: potential benefit or income that is foregone as a result of selecting one alternative over another

Examples:

- Going to cinema instead of working: direct cost is ticket 10 €, opportunity cost is 40 € one could have earned by working instead
- Storage feeding electricity into grid at 4pm: direct cost is zero, opportunity cost is 100 €/MWh storage could earn by waiting and feeding in at 8pm instead

# Terms and definitions of cost accounting

Consider some production process.

- **Fixed costs** $C_{fix}$ are the share of total costs that do not change when the produced quantity $Q$ is varied, $\frac{dC_{fix}}{dQ} = 0$.
- **Variable costs** $C_{var}(Q)$ are the share of total costs that do change when the produced quantity $Q$ is varied.
- **Total costs** are the sum of fixed and variable costs, $C(Q) = C_{fix} + C_{var}(Q)$.
- **Average costs** are the total costs per unit: $AC(Q) = \frac{C(Q)}{Q}$.
- **Marginal costs** are the costs incurred per unit for an additional unit of production; they depend on current production rate $MC(Q) = \frac{dC}{dQ}$.
- **Revenue/turnover** $p \cdot Q$ is the income from selling at price $p$.
- **Contribution margin** is the revenue minus variable costs $CM(Q) = p \cdot Q - C_{var}(Q)$, i.e. the contribution towards covering the fixed costs.

Consider a firm with 3 product lines $A, B, C$. Which should they discontinue?

| | A | B | C |
|---|---|---|---|
| Turnover | 800 | 500 | 700 |
| Variable Cost | 350 | 150 | 400 |
| Fixed Cost | 150 | 150 | 500 |
| Total Cost | 500 | 300 | 900 |
| Operating income | 300 | 200 | - 200 |
| **Overall outcome** | | **300** | |

# Total cost consideration without $C$

Discontinuing $C$ leads to a worse outcome because fixed costs still need to be paid.

| | A | B | C |
|---|---|---|---|
| Turnover | 800 | 500 | 0 |
| Variable Cost | 350 | 150 | 0 |
| Fixed Cost | 150 | 150 | 500 |
| Total Cost | 500 | 300 | 500 |
| Operating income | 300 | 200 | - 500 |
| **Overall outcome** | | 0 | |

Consider **contribution margin**: revenue/turnover minus variable costs. This is the contribution to covering the fixed costs.

| | A | B | C |
|---|---|---|---|
| Turnover | 800 | 500 | 700 |
| Variable Cost | 350 | 150 | 400 |
| Contribution margin | 450 | 350 | 300 |
| Total contribution margin | | 1100 | |
| Fixed cost | | 800 | |
| **Overall outcome** | | **300** | |

# Contribution margin: lessons

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In the **short-run** it's better to partly cover fixed costs instead of stopping production.

$\Rightarrow$ Ignore fixed costs in the short-run.

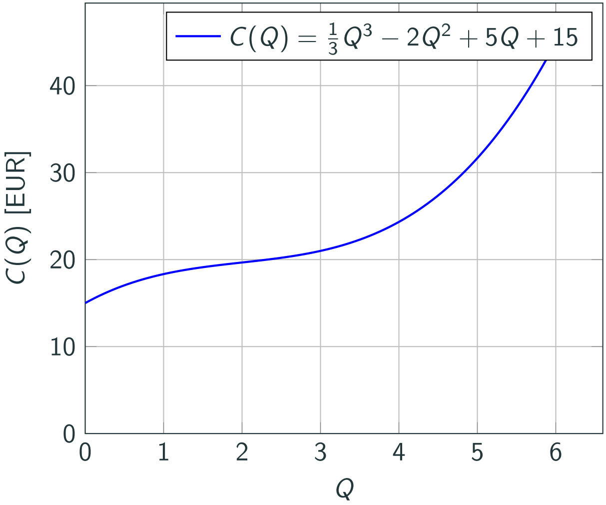
$\Rightarrow$ Maintain production as long as contribution margin (revenue minus variable costs) is positive.

Variable costs can change in the short term (gas, electricity, oil, iron ore, etc.).

In the **long-run** if the contribution margin is persistently less than fixed costs, consider scrapping plant to remove fixed costs (if investment is already paid off).

# Cost structure: cubic example

Consider a cubic total cost function $C(Q) = \frac{1}{3}Q^3 - 2Q^2 + 5Q + 15$, units EUR.



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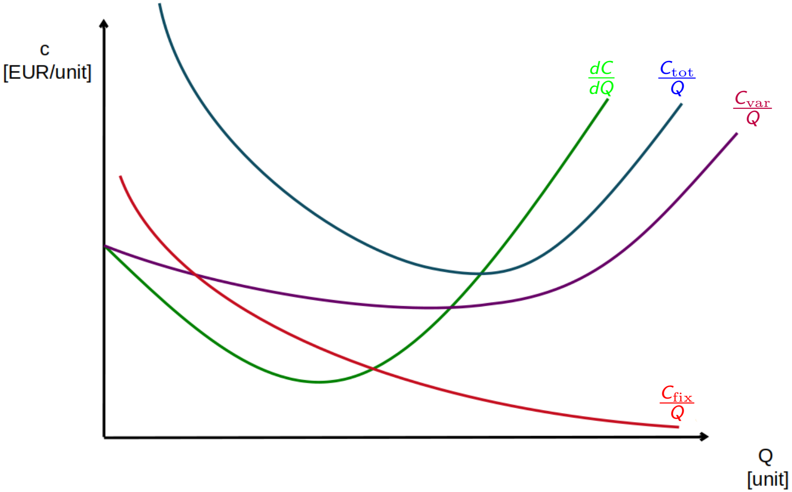


# Recall different market forms

**Perfect competition**:

- many buyers and sellers, none of whom can influence price
- goods homogeneous
- all have perfect information
- no entry or exit barriers

**Monopoly**:

- single seller who can use **market power** to influence the price

**Cournot oligopoly**:

- several sellers
- no cooperation/collusion among them

A firm wants to maximise its profits $\Pi$, i.e. revenue $p \cdot Q$ minus costs $C(Q)$, by changing its production $Q$:

$$\Pi = p \cdot Q - C(Q)$$

Looking for the maximum we find:

$$0 = \frac{d\Pi}{dQ} = \frac{d\,(p \cdot Q)}{dQ} - \frac{dC(Q)}{dQ} = \frac{dp}{dQ} \cdot Q + p - \frac{dC(Q)}{dQ}$$

For **perfect competition** the firm is a **price taker** so that it has no influence on the price $\frac{dp}{dQ} = 0$ and thus:

$$p = \frac{dC(Q)}{dQ}$$

For a given price $p$ from the market, it adjusts its output until its marginal cost $\frac{dC(Q)}{dQ}$ is equal to the price. This is a central result of microeconomics!

For a **monopoly** where the firm has market power we have to do more work since $\frac{dp}{dQ} \neq 0$.

# Production threshold and profit break-evens

From what price is production worthwhile? And from what price do we make a profit?

The **threshold for production** is the minimum price $p_{\mathrm{prod}}$ at which revenue $p \cdot Q$ covers variable costs $C_{\mathrm{var}}(Q)$. At this threshold the contribution margin is zero and we cannot cover any fixed costs. Below this threshold the production should be stopped.

To find the production threshold, find the minimum of the average variable cost:

$$0 = \frac{d\left(\frac{C_{\mathrm{var}}}{Q}\right)}{dQ} = \frac{1}{Q}\frac{dC_{\mathrm{var}}}{dQ} - \frac{1}{Q^2}C_{\mathrm{var}} \Rightarrow \frac{C_{\mathrm{var}}}{Q} = \frac{dC_{\mathrm{var}}}{dQ}$$

i.e. production threshold is when average variable cost is equal to marginal cost.

The **break-even for profit** is the minimum price $p_{\text{break}}$ at which revenue $p \cdot Q$ covers total costs $C(Q)$. At this threshold the contribution margin equals the fixed costs.

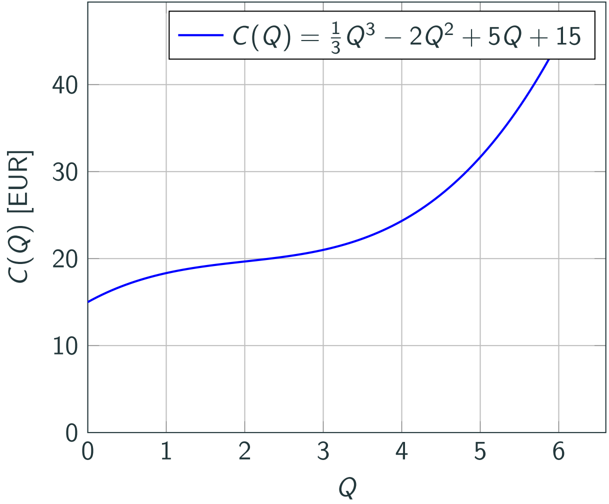
To find the break-even price, find the minimum of the average total cost:

$$0 = \frac{d\left(\frac{C}{Q}\right)}{dQ} = \frac{1}{Q}\frac{dC}{dQ} - \frac{1}{Q^2}C(Q) \Rightarrow \frac{C}{Q} = \frac{dC_{\text{var}}}{dQ}$$

i.e. profit break-even is when average cost is equal to marginal cost.

# Cost structure: cubic example

Consider a cubic total cost function $C(Q) = \frac{1}{3}Q^3 - 2Q^2 + 5Q + 15$, units EUR.



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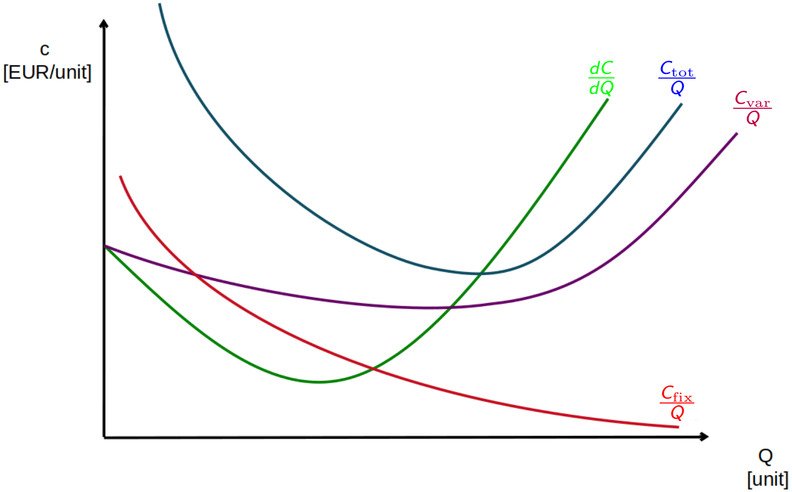


# Cost structure: cubic example

What are fixed, variable, marginal, average costs?

| | |
|---|---|
| Total costs | $C(Q) = \frac{1}{3}Q^3 - 2Q^2 + 5Q + 15$ |
| Fixed costs | $C_{\text{fix}} = 15$ |
| Variable costs | $C_{\text{var}}(Q) = \frac{1}{3}Q^3 - 2Q^2 + 5Q$ |
| Marginal costs | $\frac{dC}{dQ} = \frac{dC_{\text{var}}}{dQ} = Q^2 - 4Q + 5$ |
| Average total costs | $\frac{C}{Q} = \frac{1}{3}Q^2 - 2Q + 5 + \frac{15}{Q}$ |
| Average variable costs | $\frac{C_{\text{var}}}{Q} = \frac{1}{3}Q^2 - 2Q + 5$ |

Total costs are $C(Q) = \frac{1}{3}Q^3 - 2Q^2 + 5Q + 15$.

Suppose the producer is in a market with perfect competition and is a price taker. The market price is 10. What should the production be?

Solve:

$$10 = p = \frac{dC_{\text{var}}}{dQ} \Rightarrow 10 = Q^2 - 4Q + 5 \Rightarrow (Q-2)^2 = 9 \Rightarrow Q^* = 2 \pm 3$$

Since negative production is impossible $Q^* = 5$.

# Cost structure: cubic example

What are the production thresholds and profit break-evens?

Production threshold: solve to find production $Q_{\text{prod}}$ at threshold:

$$\frac{C_{\text{var}}}{Q} = \frac{dC_{\text{var}}}{dQ} \Rightarrow \frac{1}{3}Q^2 - 2Q + 5 = Q^2 - 4Q + 5 \Rightarrow Q_{\text{prod}} = 3$$

Here price is $p_{\text{prod}} = \frac{C_{\text{var}}(Q_{\text{prod}})}{Q_{\text{prod}}} = \frac{dC_{\text{var}}}{dQ} = 2$.

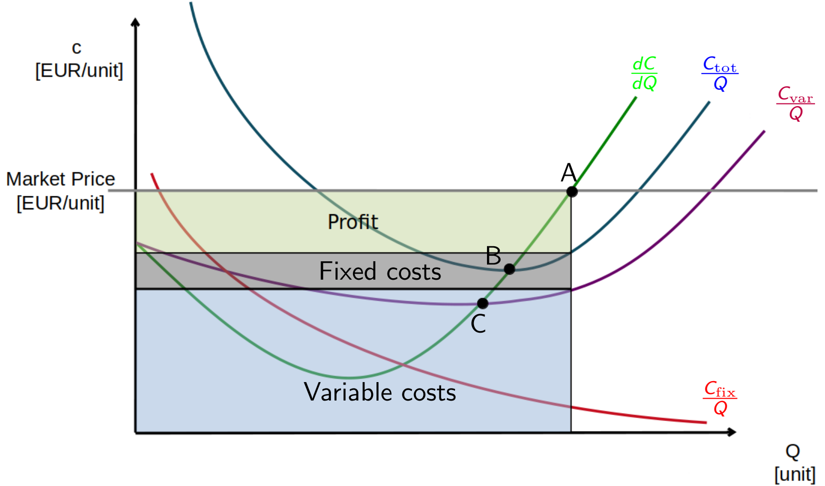
Break-even: solve to find production $Q_{\text{break}}$ at break-even:

$$\frac{C}{Q} = \frac{dC_{\text{var}}}{dQ} \Rightarrow \frac{1}{3}Q^2 - 2Q + 5 + \frac{15}{Q} = Q^2 - 4Q + 5 \Rightarrow Q_{\text{break}} = 4.25$$

Here price is $p_{\text{break}} = \frac{C(Q_{\text{break}})}{Q_{\text{break}}} = \frac{dC_{\text{var}}}{dQ} = 6.06$.

If producer is price taker in market with price $p$, it adjusts its output $Q$ so that $\frac{dC}{dQ} = p$ (A).

Production threshold (C) and profit break-even (B) are also marked.



# Monopolies

In a **monopoly** situation there is a single producer that can manipulate the price $p(Q)$ by changing its output $Q$ to maximise its profit. The price is no longer set externally.

Now the profits are given by revenue minus costs:

$$\Pi = R(Q) - C(Q) = p(Q) \cdot Q - C(Q)$$

Looking for the maximum we find:

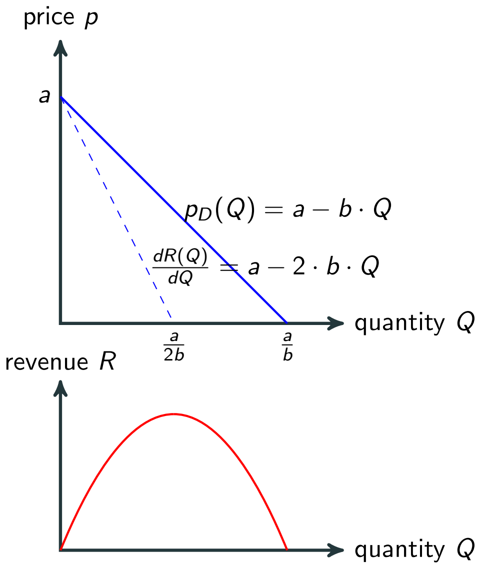
$$0 = \frac{d\Pi}{dQ} = \frac{dR(Q)}{dQ} - \frac{dC(Q)}{dQ} = \frac{d\left(p \cdot Q\right)}{dQ} - \frac{dC(Q)}{dQ} = \frac{dp}{dQ} \cdot Q + p - \frac{dC(Q)}{dQ}$$

Unlike the case of perfect competition, we now have $\frac{dp}{dQ} \neq 0$ and we find at the optimal point:

$$\frac{dC(Q)}{dQ} = \frac{dR(Q)}{dQ} = p + \frac{dp}{dQ} \cdot Q = p\left(1 + \frac{1}{\eta_{p,Q}}\right)$$

Since the price-demand elasticity is (usually) negative $\eta_{p,Q} < 0$ we can see that in a monopoly that $p > \frac{dC(Q)}{dQ}$, i.e. the price is **higher** than the marginal costs and thus higher than it would be with perfect competition.

# Special case: linear demand function




Suppose we are in a market where the consumers can be described by a linear aggregated inverse demand function:

$$p_D(Q) = a - b \cdot Q$$

The revenue $R(Q)$ is given by

$$R(Q) = p_D(Q) \cdot Q = a \cdot Q - b \cdot Q^2$$

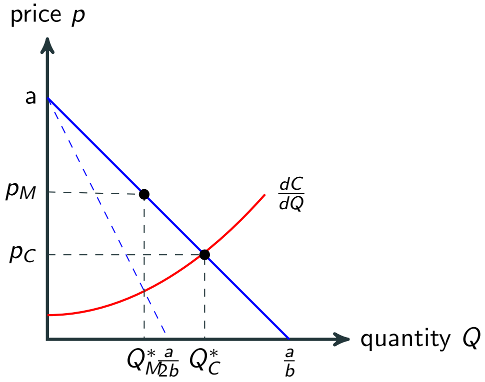
The **marginal revenue** is given by

$$\frac{dR(Q)}{dQ} = a - 2 \cdot b \cdot Q$$

The marginal revenue is linear with double the slope of the inverse demand function.

# Monopoly with linear demand function

Consider a monopoly producer with linear inverse demand function $p(Q) = a - b \cdot Q$.




For a monopoly, the optimal production level $Q_M^*$ is set when the marginal revenue equals the marginal cost:

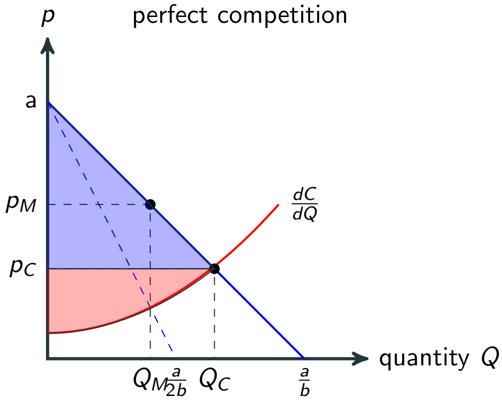
$$\frac{dR(Q)}{dQ} = a - 2 \cdot b \cdot Q = \frac{dC(Q)}{dQ}$$

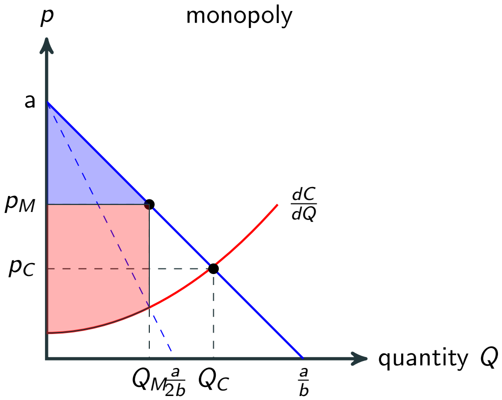
Compare this production level and the resulting price $p_M$ to the case of perfect production where the production level $Q_C^*$ and price $p_C$ are set where the inverse demand function equals the marginal cost:

$$p_D(Q) = a - b \cdot Q = \frac{dC(Q)}{dQ}$$

# Welfare in monopoly with linear demand function

Now compare the producer and consumer surpluses in the two cases. In the monopoly situation the producer increases its surplus at the expense of the consumer and the overall total welfare.

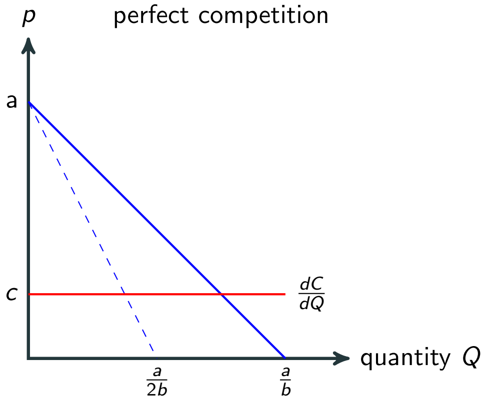


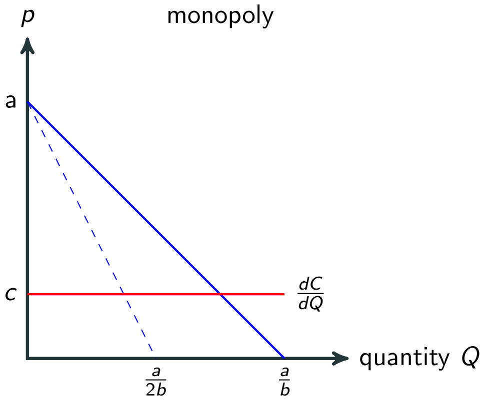



# Monopoly example: special case of linear supply costs

Now suppose that as well as a linear demand function $p_D(Q) = a - b \cdot Q$ we also have linear supply costs $C(Q) = c \cdot Q$ so that we have a constant supply function $p_S(Q) = \frac{dC}{dQ} = c$.

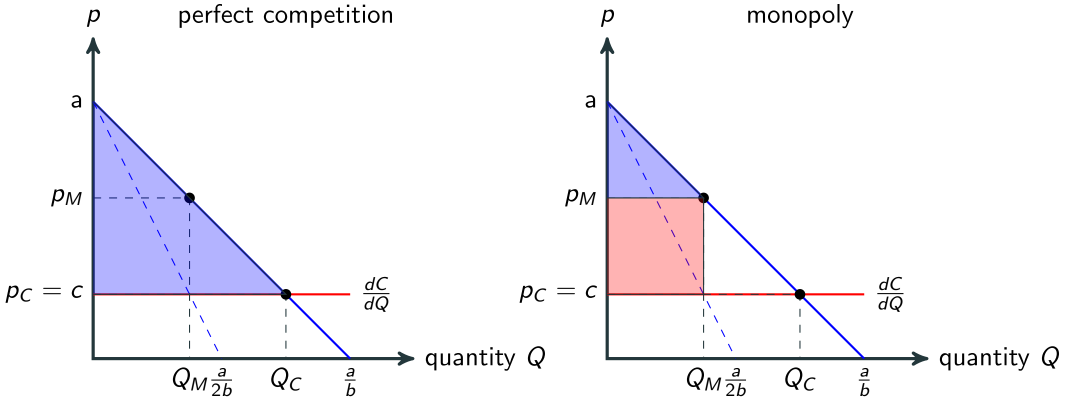
What are $p_M, p_C, Q_M, Q_C$? What about the producer and consumer surpluses?



For perfect competition $p_C = c$ and $Q_C = \frac{a-c}{b}$. Producer surplus is zero $PS = 0$, while consumer surplus is $CS = \frac{(a-c)^2}{2b}$.

For monopoly $p_M = \frac{a+c}{2}$ and $Q_M = \frac{a-c}{2b}$ (half of perfect competition). Producer surplus is now non-zero $PS = \frac{(a-c)^2}{4b}$ and $CS = \frac{(a-c)^2}{8b}$.



# Cournot oligopoly

# Cournot oligopoly

A **Cournot oligopoly** is a simplified model that interpolates between the cases of a monopoly (only one supplier) and perfect competition with many suppliers. It has the following properties:

- More than one producing firm
- All firms produce one homogeneous product (no product differentiation)
- No cooperation among firms (no collusion)
- Firms have **market power** - each firm's output decision affects the good's price
- Fixed number of firms
- Firms compete in quantities, and choose them simultaneously
- Economically rational and strategically acting firms, seeking to maximize profit given their competitors' decisions

Two identical firms produce the same good in quantities $Q_1, Q_2$ with linear supply functions and the same marginal cost:

$$C_1(Q_1) = c \cdot Q_1$$
$$C_2(Q_2) = c \cdot Q_2$$

The price is give through a linear demand function (with $a > c$):

$$p(Q_1, Q_2) = p(Q_1 + Q_2) = a - b \cdot (Q_1 + Q_2)$$

The profit functions of the two firms are given by:

$$\Pi_1(Q_1) = p(Q_1 + Q_2) \cdot Q_1 - C_1(Q_1)$$
$$\Pi_2(Q_2) = p(Q_1 + Q_2) \cdot Q_2 - C_2(Q_2)$$

Consider the maximisation of the profit $\Pi_1$ of firm 1 assuming that the production $Q_2$ of firm 2 is constant and not affected by $Q_1$:

$$\begin{aligned}\frac{d\Pi_1}{dQ_1} &= p(Q_1 + Q_2) + \frac{dp}{dQ_1} \cdot Q_1 - \frac{dC_1}{dQ_1} = a - b \cdot (Q_1 + Q_2) - bQ_1 - c \\ &= a - c - 2bQ_1 - bQ_2 = 0\end{aligned}$$

Similarly for the maximisation of the profit $\Pi_2$ of firm 2:

$$\begin{aligned}\frac{d\Pi_2}{dQ_2} &= p(Q_1 + Q_2) + \frac{dp}{dQ_2} \cdot Q_2 - \frac{dC_2}{dQ_2} = a - b \cdot (Q_1 + Q_2) - bQ_2 - c \\ &= a - c - bQ_1 - 2bQ_2 = 0\end{aligned}$$

From the second equation we can substitute $bQ_1 = a - c - 2bQ_2$ into the first to get:

$$Q_2^* = \frac{a - c}{3b} = Q_1^*$$

(note the symmetry between the identical firms) and as a result $p^* = \frac{a+2c}{3}$.

# Interpolation from monopoly to polipoly

Using the Cournot model we can interpolate between a monopoly $N = 1$ and perfect competition for the case of linear supply costs $C_i(Q_i) = c \cdot Q_i$ and linear demand function $p_D(\sum_i Q) = a - b \cdot \sum_i Q_i$:

| market type | $N$ | price | sales |
|---|---|---|---|
| monopoly | 1 | $p = \frac{a+c}{2}$ | $Q = \frac{a-c}{2b}$ |
| duopoly | 2 | $p = \frac{a+2c}{3}$ | $Q_i = \frac{a-c}{3b}$ |
| oligopoly | $N$ | $p = \frac{a+Nc}{N+1}$ | $Q_i = \frac{a-c}{b(N+1)}$ |
| polipoly | $N \to \infty$ | $p = c$ | $\sum_i Q_i = \frac{a-c}{b}$ |