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Energy Economics, Winter Semester 2021-2
Lecture 3: Basics of Microeconomics

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Types of Markets

Monopoly versus perfect competition

A **market** is where a group of sellers and a group of buyers of a particular good or service come together for exchange.

For **perfect competition** to apply we need:

- many buyers and many sellers so nobody can influence the price, i.e. all actors are **price takers**
- goods are exactly the same, i.e. **homogeneous**
- all actors have perfect information
- no entry or exit barriers

In a **monopoly** there is a **single seller**:

- Seller is sole producer and can influence the price of its output
- Seller has **market power**: the ability to maintain a price above the price under competition

Monopoly versus perfect competition

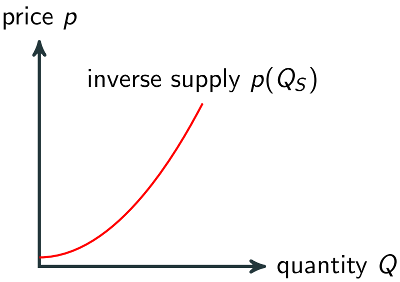
	one seller	few sellers	many sellers
one buyer	Bilateral monopoly		Monopsony (buyer's monopoly)
few buyers	Oligopolistic market structures		
many buyers	Seller's monopoly		Perfect competition

Perfect Competition

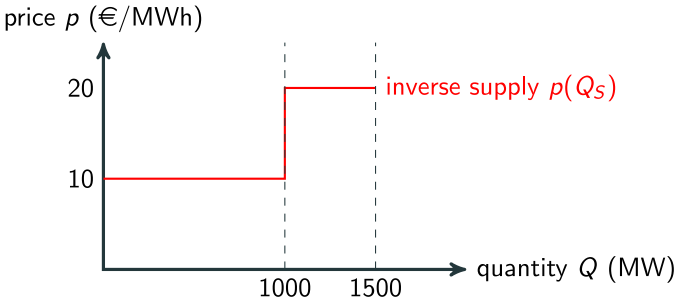
Inverse supply functions

Each producing firm has its own **inverse supply function** which shows at which price p it is worthwhile to sell a given quantity Q_S. It indicates the **marginal cost** of the next unit of production at a given production level of Q_S units. Generally: the higher the quantity, the higher the price.

(It's the *inverse* because the supply function is the quantity as a function of price $Q_S(p)$.)

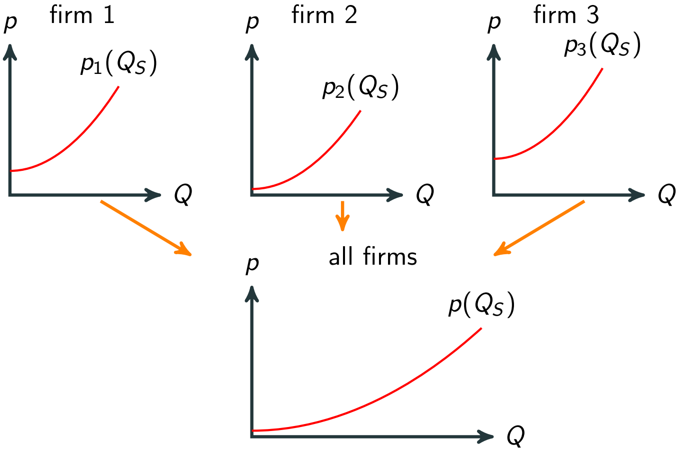


A generation firm has two coal plants. The new plant can produce 1000 MW at a cost of 10 €/MWh. An older less efficient plant can produce only 500 MW at a cost of 20 €/MWh.



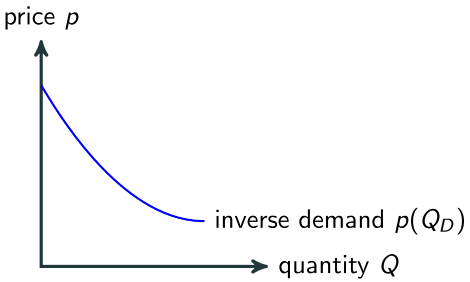
Aggregation of inverse supply functions

If many firms are active, we can sort and aggregate their inverse supply functions into a single supply function for the whole market.



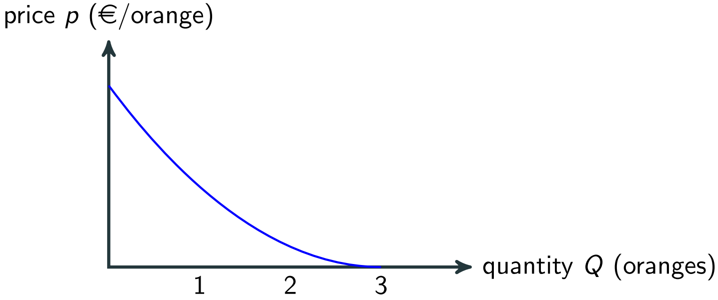
Each consumer has its own **inverse demand function** which shows which price p it is prepared to pay for a given quantity Q_D. It indicates the **willingness to pay** or **marginal utility** of the next unit of consumption at a given consumption level of Q_D units. Generally: the higher the quantity, the lower the price.

(It's the *inverse* because the demand function is the quantity as a function of price $Q_D(p)$.)



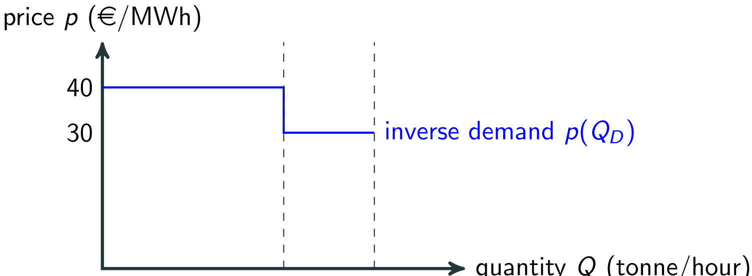
Inverse demand function: example

After sport I'm hungry and thirsty. I visit a stand selling oranges. The first orange is worth a lot to me. It partially satisfies my hunger and thirst, so the second is worth less to me. By the 3rd orange, I'm full.



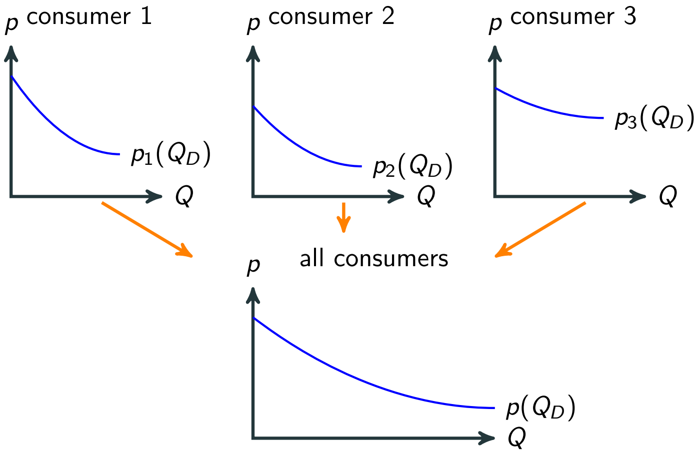
Electrolysis is used to produce aluminium from bauxite ore. On the open market aluminium sells for €1200/tonne. The cost of materials and labour is €600/tonne, which leaves €600/tonne to pay for electricity. A firm has two electrolysis units. A newer efficient one consumes 15 MWh/tonne, while an older inefficient one consumes 20 MWh/tonne. How much are they willing to pay for the electricity?

New: $\frac{600€/\mathrm{t}}{15\mathrm{MWh/t}} = 40€/\mathrm{MWh}$, old $\frac{600€/\mathrm{t}}{20\mathrm{MWh/t}} = 30€/\mathrm{MWh}$.



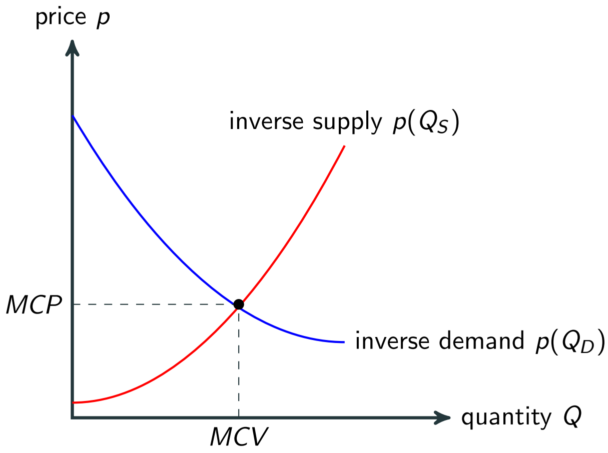
Aggregation of inverse demand functions

If many consumers are active, we can sort and aggregate their inverse demand functions into a single demand function for the whole market.

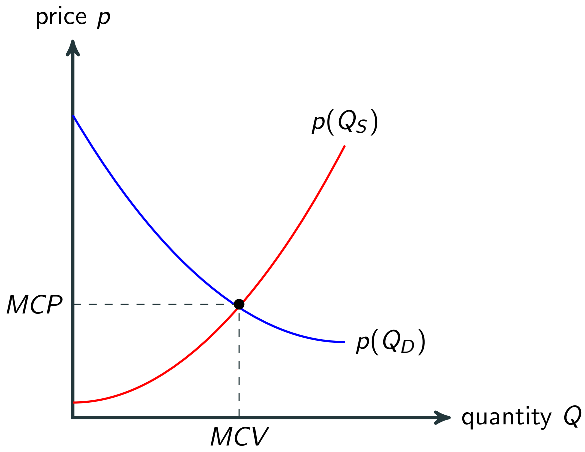


Market clearing price and volume

In a competitive market the **market clearing price** (MCP) and the **market clearing volume** (MCV) are set by the intersection of the inverse supply and demand functions.



Market clearing price and volume: important properties



- Every actor sees the same price.
- The price arises **decentrally** from the interaction of supply and demand curves.
- The same amount is supplied as is consumed.
- Since there are many suppliers and consumers, no single actor can influence the price (they are all **price takers**).
- The price is higher than the costs of each supplier whose offer is taken.
- The price is lower than the willingness to pay of each consumer whose bid is taken.

Market clearing price and volume: simple example

Consider a market with:

Aggregated inverse demand curve $p_D(Q_D) = 28 - 4Q_D$.

Aggregated inverse supply curve $p_S(Q_S) = 1 + 5Q_S$.

Q: What is the market clearing price and volume?

Solve

$$28 - 4Q = 1 + 5Q$$

Find $Q^* = 3, p^* = 16$.

Reaction to increase in demand

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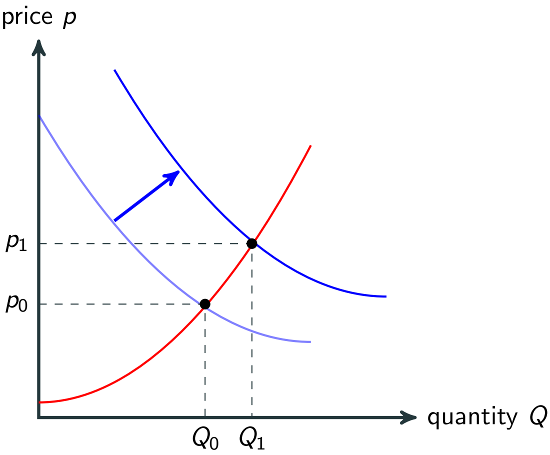
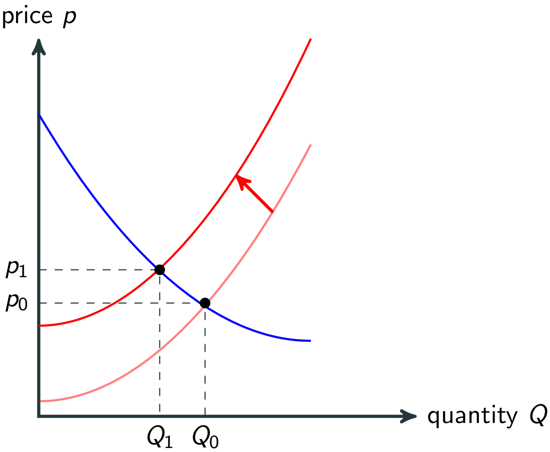
If demand increases in volume and/or willingness to pay, the price goes up.



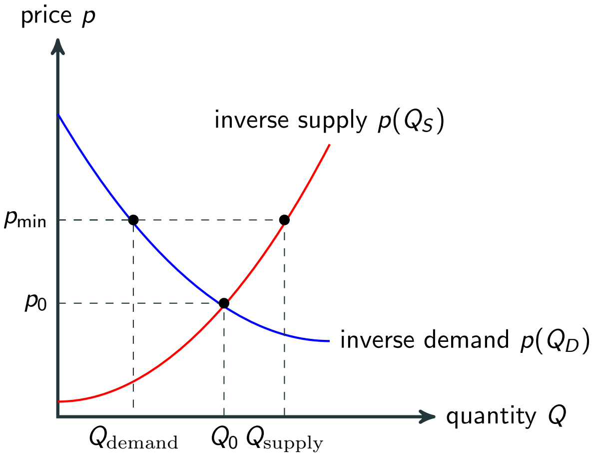
Reaction to increase in supply cost

If supply becomes more expensive or decreases in volume, the price goes up.



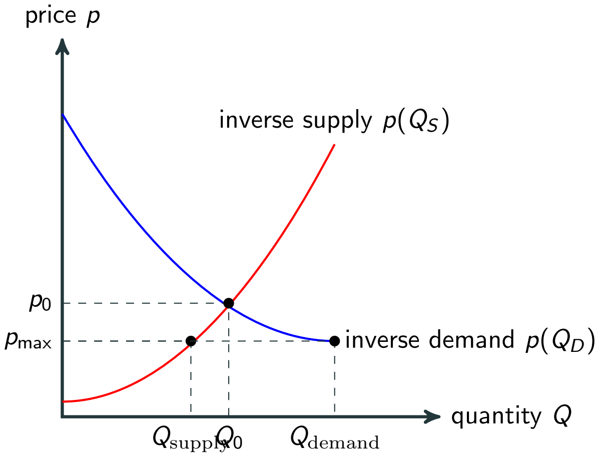
Reaction to state regulation: minimum price

A minimum price or floor price can lead to lower demand and/or excess supply. Cf. butter mountains and milk lakes in EU in past; minimum wages.



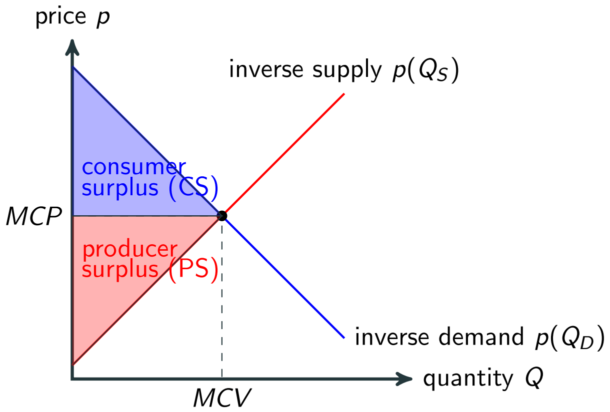
Reaction to state regulation: maximum price

A maximum price or price cap can lead to lower supply and/or excess demand. Cf. rent caps, energy price caps.



The **consumer surplus** is the total amount consumers are willing to pay minus what they actually pay. The **producer surplus** in the total revenue for producers minus their actual costs.

The **total surplus** or **total welfare** $TS = CS + PS$ indicates the degree of efficiency of resource allocation.



Consumer and producer surpluses: example

Consider the previous example:

Aggregated inverse demand curve $p_D(Q_D) = 28 - 4Q_D$.

Aggregated inverse supply curve $p_S(Q_S) = 1 + 5Q_S$.

MCV and MCP are $Q^* = 3$, $p^* = 16$.

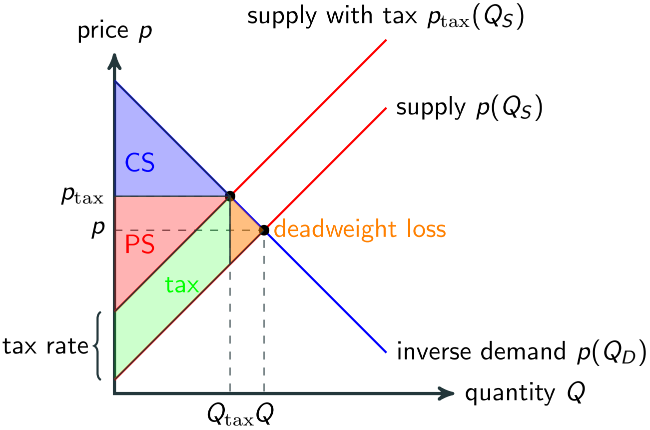
Q: What are the consumer and producer surpluses?

Consumer surplus: $CS = 0.5 * 3 * (28 - 16) = 18$.

Producer surplus: $PS = 0.5 * 3 * (16 - 1) = 22.5$.

Total surplus: $TS = CS + PS = 18 + 22.5 = 40.5$.

Consumer and producer surpluses: response to tax



Market clearing price and volume with tax: simple example

Consider a market with:

Aggregated inverse demand curve $p_D(Q_D) = 28 - 4Q_D$.

Aggregated inverse supply curve $p_S(Q_S) = 1 + 5Q_S$.

Suppose we tax the product at a rate of 9€/unit. What is the market clearing price and volume now?

Solve

$$28 - 4Q = 10 + 5Q$$

Find $Q^* = 2, p^* = 30$.

Market clearing price and volume with tax: simple example

Consider a market with:

Aggregated inverse demand curve $p_D(Q_D) = 28 - 4Q_D$.

Aggregated inverse supply curve $p_S(Q_S) = 1 + 5Q_S$.

Suppose we tax the product at a rate of 9€/unit. How do the surpluses change?

Consumer surplus: $CS_{\text{tax}} = 0.5 * 2 * (28 - 20) = 8$.

Producer surplus: $PS_{\text{tax}} = 0.5 * 2 * (20 - 10) = 10$.

Total tax revenue: $TR_{\text{tax}} = 2 * 9 = 18$.

Total surplus includes tax revenue: $TS_{\text{tax}} = CS + PS + TR = 36$.

Deadweight loss: $TS - TS_{\text{tax}} = 40.5 - 18 - 18 = 4.5$.

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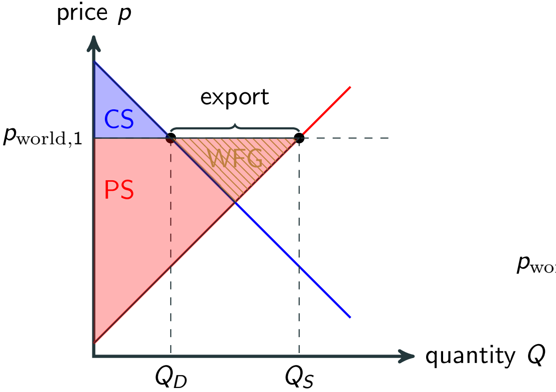
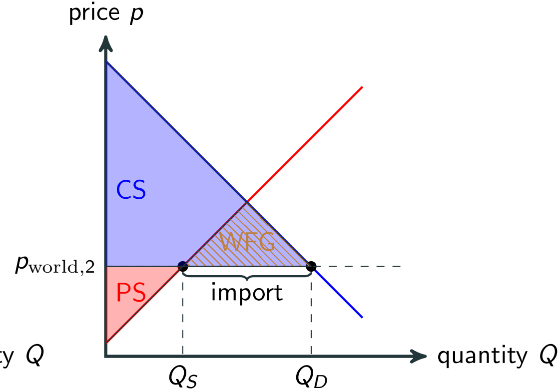
Trade between 2 regions always leads to a **welfare gain** (WFG) in each country, but can have a strong influence on the distribution between the consumer and producer surpluses.





Elasticity

Price elasticity

Elasticity is a is a measure of how much buyers and sellers respond to changes in market conditions.

Price elasticity measures the response to price changes. Price elasticity of demand $\eta_{p,Q}$ is a measure of how much the quantity demanded of a good responds to a change in the price of that good.

$$\text{Price elasticity of demand} = \frac{\%\text{ change in quantity demanded}}{\%\text{ change in price}}$$

$$\eta_{p,Q} = \frac{\frac{dQ}{Q}}{\frac{dp}{p}} = \frac{dQ}{dp} \cdot \frac{p}{Q}$$

Elastic versus inelastic demand

Demand-price elasticity is generally negative: the higher the price, the lower the demand. The magnitude of the elasticity allows it to be classified:

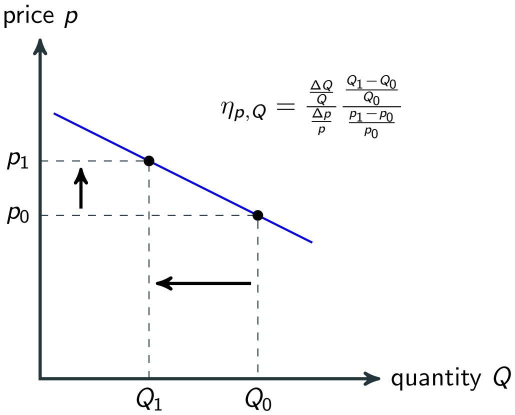
$-\infty < \eta_{p,Q} \leq -1$ **elastic demand** (big change in demand for small change in price)

$\eta_{p,Q} = -1$ **isoelastic demand** (same % change in demand for % change in price)

$-1 < \eta_{p,Q} \leq 0$ **inelastic demand** (small change in demand for large change in price)

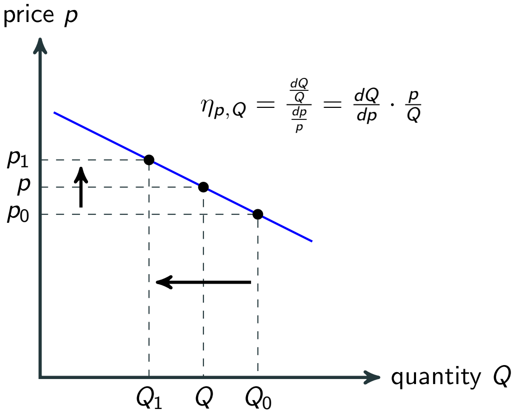
If demand does not respond at all to prices, $\eta_{p,Q} = 0$, it is **perfectly inelastic**.

The **arc elasticity** can be measured if you know how the demand responds to a specific price increase from p_0 to p_1.



Point elasticity

The **point elasticity** is the infinitesimal version if you can precisely calculate the derivative.



Examples of price-demand elasticity

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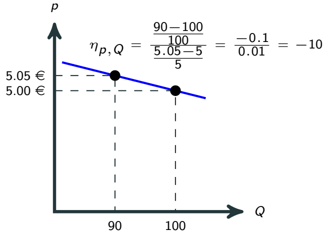
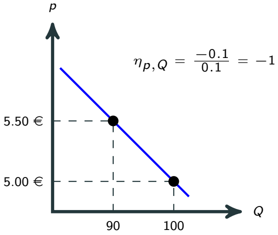
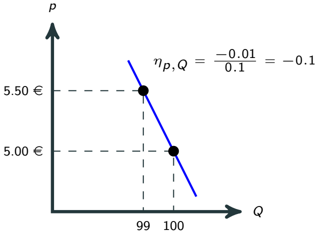
Consider the following arc elasticities:



very elastic (small change in price ⇒ big change in demand)



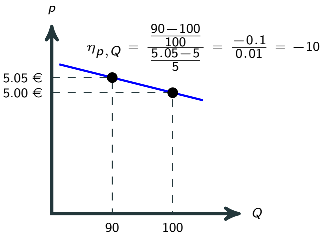
isoelastic (change in price ⇒ propotional change in demand)



very inelastic (large change in price ⇒ small change in demand)

Maximising revenue

Suppose a supplier has the market power to manipulate the price, and they have no marginal costs. How to maximise the **revenue** $R(p) = p \cdot Q(p)$? Consider effect of price increase:

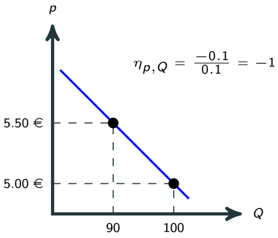


elastic

$R(p) = 5 \cdot 100 = 500$

$\rightarrow 5.05 \cdot 90 = 454.50$

$\Rightarrow$ better to drop the price

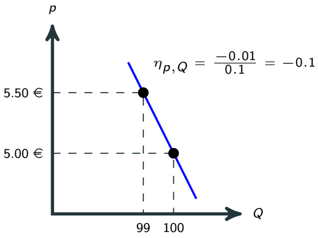


isoelastic

$R(p) = 5 \cdot 100 = 500$

$\rightarrow 5.5 \cdot 90 = 495$

$\Rightarrow$ leave price the same



inelastic

$R(p) = 5 \cdot 100 = 500$

$\rightarrow 5.5 \cdot 99 = 544.50$

$\Rightarrow$ raise the price

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Suppose a supplier has the market power to manipulate the price, and they have no marginal costs. How would they maximise the **revenue** $R(p) = p \cdot Q(p)$?

We can also demonstrate this mathematically by maximising $R(p)$:

$$0 = \frac{dR}{dp} = \frac{d(p \cdot Q(p))}{dp} = p \cdot \frac{dQ(p)}{dp} + Q(p) = Q\left(\frac{p}{Q}\frac{dQ(p)}{dp} + 1\right) = Q(\eta_{p,Q} + 1)$$

So the revenue is maximised when we reach the isoelastic point $\eta_{p,Q} = -1$.

Electricity demand: typically inelastic

Demand for electricity is largely inelastic.

General reasons for inelasticity:

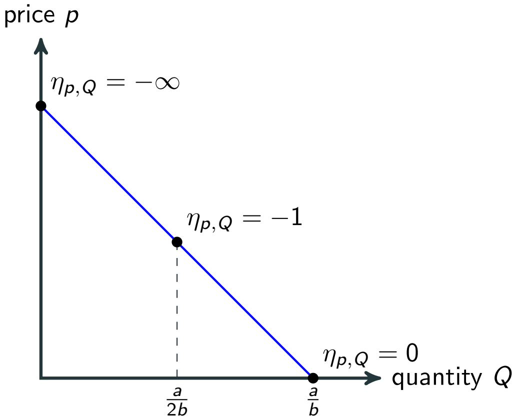
- Consumers do not perceive the price changes (hidden in monthly bills)
- Lack of substitutes (e.g. for heating: gas, for lighting: gas lamps, for communication: pigeon?, for computation: abacus?)
- Switching to alternative products (substitutes) is cumbersome

Distinguish between **short-run elasticity** and **long-run elasticity**. Long-run elasticity allows consumers time (e.g. years) to become more aware of and purchase alternatives. In the long-run demand tends to be more elastic than the short-run.

E.g. if petrol prices remain high over many years, consumers may be more likely to switch to electric vehicles.

Beware: slope is not the same as elasticity

The slope of the inverse demand curve is **not the same** as the elasticity.



For a linear inverse demand function:

$$p(Q) = a - bQ$$

for $a, b > 0$ we have the demand function:

$$Q(p) = \frac{a-p}{b}$$

The elasticity varies with Q:

$$\eta_{p,Q} = \frac{p}{Q}\frac{dQ}{dp} = 1 - \frac{a}{bQ}$$

Positive price-demand elasticity

For some goods, the price-demand elasticity can become **positive**, i.e. rising prices lead to rising demand.

- **Veblen or Snob effect**: A product becomes more attractive the more expensive it is (e.g. exclusive clubs in London, whisky, cigars, Renoirs).
- **Quality effect**: If quality is hard to assess, price is used as a quality indicator (e.g. wine).

If the price of one good has an effect on the demand of another good, this indirect elasticity is called a **cross elasticity**. For example, if the price of one good p_1 influences the sales of another Q_2 then the cross elasticity is given by:

$$\eta_{p_1,Q_2} = \frac{p_1}{Q_2}\frac{dQ_2}{dp_1}$$

- Example of **negative** cross-price-elasticity: rising petrol prices leave to sinking demand for cars. Petrol and cars are **complementary goods**.
- Example of **positive** cross-price-elasticity: rising butter prices lead to rising demand for margarine. Butter and margarine are **substitute goods**.

The price for electricity is 0.2€/kWh. The demand function of a private household (per month) is given by:

$$Q_D(p) = 625 - 625p$$

where the units of Q_D are kWh and of p are €/kWh.

a) How much electricity does the single household consume per month? How much does it pay?

$$Q_D(p = 0.2) = 625 - 0.2 * 625 = 500$$

Consumption: 500 kWh.

$$p \cdot Q_D = 0.2 * 500 = 100$$

Pay per month: 100 €/m.

Elasticity: simple example

The price for electricity is 0.2€/kWh. The demand function of a private household (per month) is given by:

$$Q_D(p) = 625 - 625p$$

where the units of Q_D are kWh and of p are €/kWh.

b) What is the price elasticity at this point? Is it elastic or inelastic?

$$\begin{aligned} \eta_{p,Q} &= \frac{dQ}{dp} \cdot \frac{p}{Q} \\ &= -625 \cdot \frac{0.2}{500} \\ &= -0.25 \end{aligned}$$

The demand is inelastic.

The price for electricity is 0.2€/kWh. The demand function of a private household (per month) is given by:

$$Q_D(p) = 625 - 625p$$

where the units of Q_D are kWh and of p are €/kWh.

c) How does the household react if the price doubles? What are the demand and elasticity now?

$$Q_D(p = 0.4) = 625 - 0.4 * 625 = 375$$

$$\begin{aligned}\eta_{p,Q} &= \frac{dQ}{dp} \cdot \frac{p}{Q} \\ &= -625 \cdot \frac{0.4}{375} \\ &= -0.67\end{aligned}$$

The demand is still inelastic, but a little more elastic i.e. a little more price-responsive.

The price for electricity is 0.2€/kWh. The demand function of a private household (per month) is given by:

$$Q_D(p) = 625 - 625p$$

where the units of Q_D are kWh and of p are €/kWh.

d) At what price does the demand become isoelastic?

Solve

$$\begin{aligned} -1 &= \eta_{p,Q} \\ &= \frac{dQ}{dp} \cdot \frac{p}{Q} \\ &= -625 \cdot \frac{p}{625 - 625p} \end{aligned}$$

We find $p = 0.5$ €/kWh and

$$Q_D(p = 0.5) = 625 - 0.5 * 625 = 312.5$$

Production decisions in different types of market

Supply side: direct versus opportunity costs

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The price at which a seller is willing to sell their goods is determined by their costs of production:

- **Explicit, direct costs**: out-of-pocket expenses, money actually paid e.g. for wages, materials, energy
- **Opportunity costs**: potential benefit or income that is foregone as a result of selecting one alternative over another

Examples:

- Going to cinema instead of working: direct cost is ticket 10 €, opportunity cost is 40 € I could have earned by working instead
- Storage feeding electricity into grid at 4pm: direct cost is zero, opportunity cost is 100 €/kWh storage could earn by waiting and feeding in at 8pm instead

Terms and definitions of cost accounting

Consider some production process.

- **Fixed costs** C_{fix} are the share of total costs that do not change when the produced quantity Q is varied, $\frac{dC_{fix}}{dQ} = 0$.
- **Variable costs** $C_{var}(Q)$ are the share of total costs that do change when the produced quantity Q is varied.
- **Total costs** are the sum of fixed and variable costs, $C(Q) = C_{fix} + C_{var}(Q)$.
- **Average costs** are the total costs per unit: $AC(Q) = \frac{C(Q)}{Q}$.
- **Marginal costs** are the costs incurred per unit for an additional unit of production; they depend on current production rate $MC(Q) = \frac{dC}{dQ}$.
- **Revenue/turnover** $p \cdot Q$ is the income from selling at price p.
- **Contribution margin** is the selling price minus variable cost per unit $CM(Q) = p \cdot Q - C_{var}(Q)$, i.e. the contribution towards covering the fixed costs.

Total cost consideration

Consider a firm with 3 product lines A, B, C. Which should they discontinue?

	A	B	C
Turnover	800	500	700
Variable Cost	350	150	400
Fixed Cost	150	150	500
Total Cost	500	300	900
Operating income	300	200	- 200
Overall outcome		**300**	

Total cost consideration without C

Discontinuing C leads to a worse outcome because fixed costs still need to be paid.

	A	B	C
Turnover	800	500	0
Variable Cost	350	150	0
Fixed Cost	150	150	500
Total Cost	500	300	500
Operating income	300	200	- 500
Overall outcome		0	

Contribution margin

Consider **contribution margin**: revenue/turnover minus variable costs. This is the contribution to covering the fixed costs.

	A	B	C
Turnover	800	500	700
Variable Cost	350	150	400
Contribution margin	450	350	300
Total contribution margin		1100	
Fixed cost		800	
Overall outcome		**300**	

Contribution margin: lessons

In the **short-run** it's better to partly cover fixed costs instead of stopping production.

$\Rightarrow$ Ignore fixed costs in the short-run.

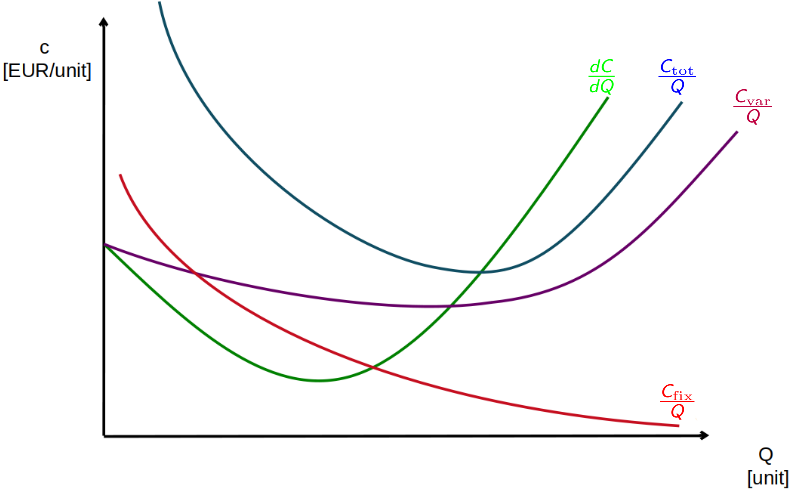
$\Rightarrow$ Maintain production as long as contribution margin (revenue minus variable costs) is positive.

Variable costs can change in the short term (gas, electricity, oil, iron ore, etc.).

In the **long-run** if the contribution margin is persistently less than fixed costs, consider scrapping plant to remove fixed costs (if investment is already paid off).

Cost structure: cubic example

Consider a cubic total cost function $C(Q) = \frac{1}{3}Q^3 - 2Q^2 + 5Q + 15$.



Recall different market forms

Perfect competition:

- many buyers and sellers, none of whom can influence price
- goods homogeneous
- all have perfect information
- no entry or exit barriers

Monopoly:

- single seller who can use **market power** to influence the price

Cournot oligopoly:

- several sellers
- no cooperation/collusion among them

A firm wants to maximise its profits Π, i.e. revenue $p \cdot Q$ minus costs $C(Q)$, by changing its production Q:

$$\Pi = p \cdot Q - C(Q)$$

Looking for the maximum we find:

$$0 = \frac{d\Pi}{dQ} = \frac{d\,(p \cdot Q)}{dQ} - \frac{dC(Q)}{dQ} = \frac{dp}{dQ} \cdot Q + p - \frac{dC(Q)}{dQ}$$

For **perfect competition** the firm is a **price taker** so that it has no influence on the price $\frac{dp}{dQ} = 0$ and thus:

$$p = \frac{dC(Q)}{dQ}$$

For a given price p from the market, it adjusts its output until its marginal cost $\frac{dC(Q)}{dQ}$ is equal to the price. This is a central result of microeconomics!

For a **monopoly** where the firm has market power we have to do more work since $\frac{dp}{dQ} \neq 0$.

Production threshold and profit break-evens

From what price is production worthwhile? And from what price do we make a profit?

The **threshold for production** is the point at which the price is above the average variable cost.

Minimum of average variable cost:

$$0 = \frac{d\left(\frac{C_{\mathrm{var}}}{Q}\right)}{dQ} = \frac{1}{Q}\frac{dC_{\mathrm{var}}}{dQ} - \frac{1}{Q^2}C_{\mathrm{var}} \Rightarrow \frac{C_{\mathrm{var}}}{Q} = \frac{dC_{\mathrm{var}}}{dQ}$$

i.e. production threshold is when average variable cost is equal to marginal cost.

The **break-even for profit** is the point at which the price is above the average total cost.

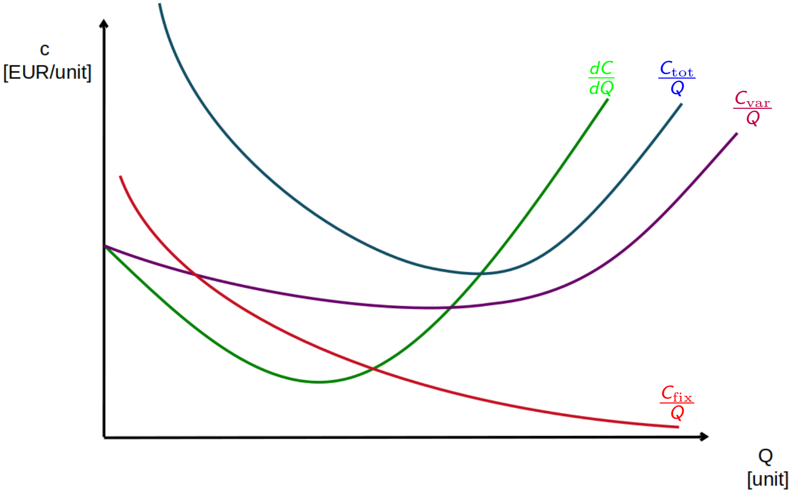
Minimum of average total cost:

$$0 = \frac{d\left(\frac{C}{Q}\right)}{dQ} = \frac{1}{Q}\frac{dC}{dQ} - \frac{1}{Q^2}C(Q) \Rightarrow \frac{C}{Q} = \frac{dC_{\text{var}}}{dQ}$$

i.e. profit break-even is when average cost is equal to marginal cost.

Cost structure: cubic example

Consider a cubic total cost function $C(Q) = \frac{1}{3}Q^3 - 2Q^2 + 5Q + 15$.



Cost structure: cubic example

What are fixed, variable, marginal, average costs?

Total costs	$C(Q) = \frac{1}{3}Q^3 - 2Q^2 + 5Q + 15$
Fixed costs	$C_{\text{fix}} = 15$
Variable costs	$C_{\text{var}}(Q) = \frac{1}{3}Q^3 - 2Q^2 + 5Q$
Marginal costs	$\frac{dC}{dQ} = \frac{dC_{\text{var}}}{dQ} = Q^2 - 4Q + 5$
Average total costs	$\frac{C}{Q} = \frac{1}{3}Q^2 - 2Q + 5 + \frac{15}{Q}$
Average variable costs	$\frac{C_{\text{var}}}{Q} = \frac{1}{3}Q^2 - 2Q + 5$

Cost structure: cubic example

Total costs are $C(Q) = \frac{1}{3}Q^3 - 2Q^2 + 5Q + 15$.

Suppose the producer is in a market with perfect competition and is a price taker. The market price is 10. What should the production be?

Solve:

$$10 = p = \frac{dC_{\mathrm{var}}}{dQ} \Rightarrow 10 = Q^2 - 4Q + 5 \Rightarrow (Q-2)^2 = 9 \Rightarrow Q^* = 2 \pm 3$$

Since negative production is impossible $Q^* = 5$.

Cost structure: cubic example

What are the production thresholds and profit break-evens?

Production threshold: solve to find production Q^* at threshold:

$$\frac{C_{\mathrm{var}}}{Q} = \frac{dC_{\mathrm{var}}}{dQ} \Rightarrow \frac{1}{3}Q^2 - 2Q + 5 = Q^2 - 4Q + 5 \Rightarrow Q^* = 3$$

Here price is $MC = \frac{dC_{\mathrm{var}}}{dQ} = 2$.

Break-even: solve to find production Q^* at break-even:

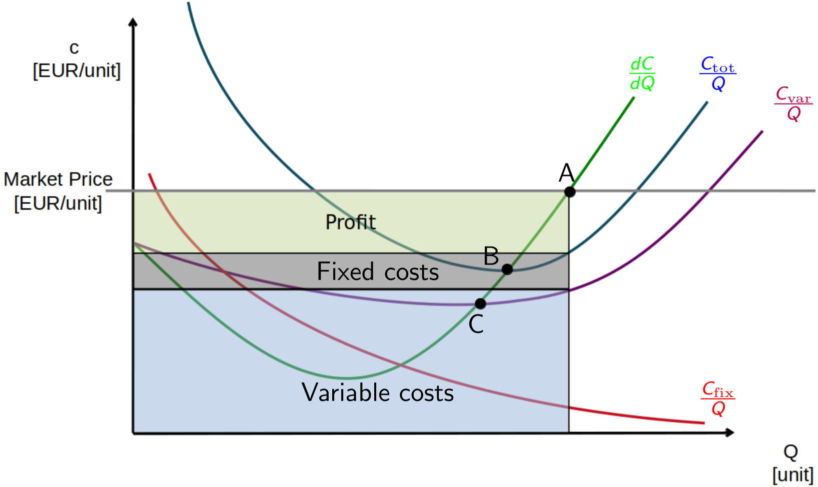
$$\frac{C}{Q} = \frac{dC_{\mathrm{var}}}{dQ} \Rightarrow \frac{1}{3}Q^2 - 2Q + 5 + \frac{15}{Q} = Q^2 - 4Q + 5 \Rightarrow Q^* = 4.25$$

Here price is $MC = \frac{dC_{\mathrm{var}}}{dQ} = 6.06$.

Cost structure: cubic example

If producer is price taker in market with price p, it adjusts its output Q so that $\frac{dC}{dQ} = p$ (A).

Production threshold (C) and profit break-even (B) are also marked.



Monopolies

Profit-maximisation in a monopoly

Technische Universität Berlin

In a **monopoly** situation there is a single producer that can manipulate the price $p(Q)$ by changing its output Q to maximise its profit. The price is no longer set externally.

Now the profits are given by revenue minus costs:

$$\Pi = R(Q) - C(Q) = p(Q) \cdot Q - C(Q)$$

Looking for the maximum we find:

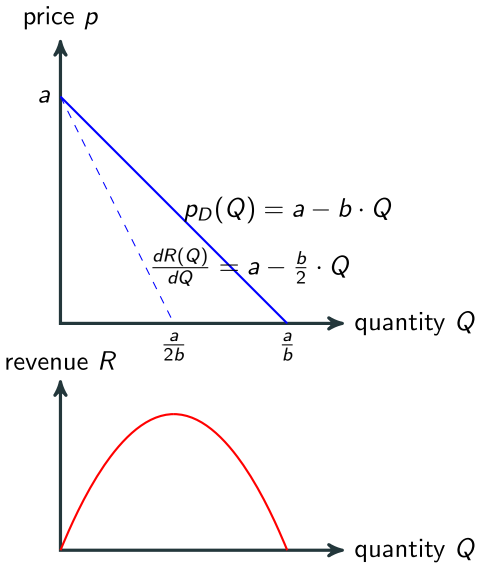
$$0 = \frac{d\Pi}{dQ} = \frac{dR(Q)}{dQ} - \frac{dC(Q)}{dQ} = \frac{d\left(p \cdot Q\right)}{dQ} - \frac{dC(Q)}{dQ} = \frac{dp}{dQ} \cdot Q + p - \frac{dC(Q)}{dQ}$$

Unlike the case of perfect competition, we now have $\frac{dp}{dQ} \neq 0$ and we find at the optimal point:

$$\frac{dC(Q)}{dQ} = \frac{dR(Q)}{dQ} = p + \frac{dp}{dQ} \cdot Q = p\left(1 + \frac{1}{\eta_{p,Q}}\right)$$

Since the price-demand elasticity is (usually) negative $\eta_{p,Q} < 0$ we can see that in a monopoly that $p > \frac{dC(Q)}{dQ}$, i.e. the price is **higher** than the marginal costs and thus higher than it would be with perfect competition.

Special case: linear demand function



Suppose we are in a market where the consumers can be described by a linear aggregated inverse demand function:

$$p_D(Q) = a - b \cdot Q$$

The revenue $R(Q)$ is given by

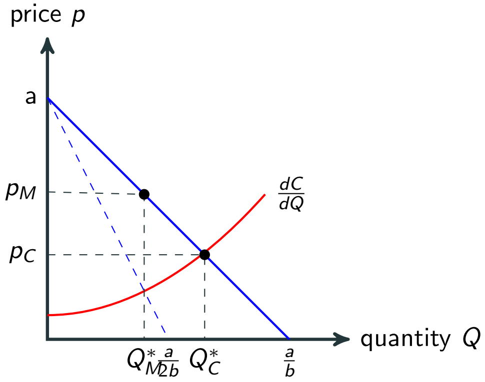
$$R(Q) = p_D(Q) \cdot Q = a \cdot Q - b \cdot Q^2$$

The **marginal revenue** is given by

$$\frac{dR(Q)}{dQ} = a - 2 \cdot b \cdot Q$$

The marginal revenue is linear with double the slope of the inverse demand function.

Consider a monopoly producer with linear inverse demand function $p(Q) = a - b \cdot Q$.



For a monopoly, the optimal production level Q_M^* is set when the marginal revenue equals the marginal cost:

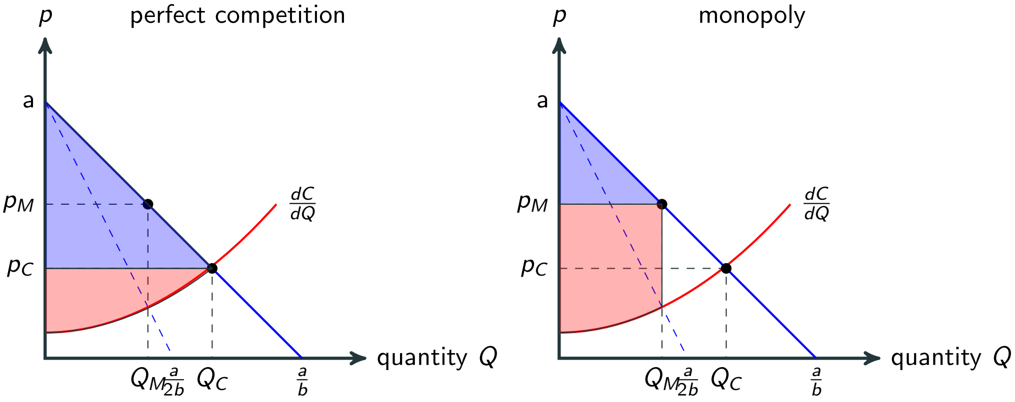
$$\frac{dR(Q)}{dQ} = a - 2 \cdot b \cdot Q = \frac{dC(Q)}{dQ}$$

Compare this production level and the resultign price p_M to the case of perfect production where the production level Q_C^* and price p_C are set where the inverse demand function equals the marginal cost:

$$p_D(Q) = a - b \cdot Q = \frac{dC(Q)}{dQ}$$

Welfare in monopoly with linear demand function

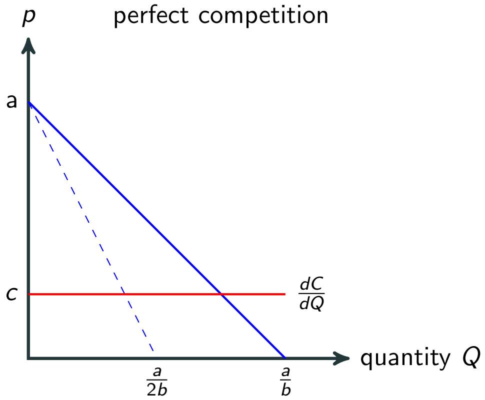
Now compare the producer and consumer surpluses in the two cases. In the monopoly situation the producer increases its surplus at the expense of the consumer and the overall total welfare.

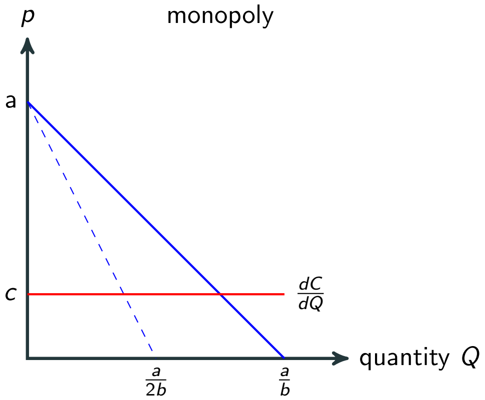


Monopoly example: special case of linear supply costs

Now suppose that as well as a linear demand function $p_D(Q) = a - b \cdot Q$ we also have linear supply costs $C(Q) = c \cdot Q$ so that we have a constant supply function $p_S(Q) = \frac{dC}{dQ} = c$.

What are p_M, p_C, Q_M, Q_C? What about the producer and consumer surpluses?

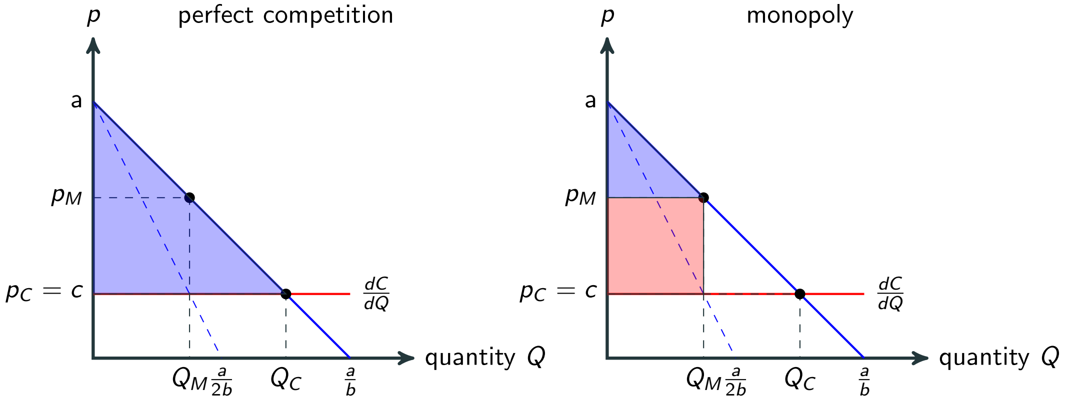




Monopoly example: special case of linear supply costs

For perfect competition $p_C = c$ and $Q_C = \frac{a-c}{b}$. Producer surplus is zero $PS = 0$, while consumer surplus is $CS = \frac{(a-c)^2}{2b}$.

For monopoly $p_M = \frac{a+c}{2}$ and $Q_M = \frac{a-c}{2b}$ (half of perfect competition). Producer surplus is now non-zero $PS = \frac{(a-c)^2}{4b}$ and $CS = \frac{(a-c)^2}{8b}$.



Cournot oligopoly

Cournot oligopoly

A **Cournot oligopoly** is a simplified model that interpolates between the cases of a monopoly (only one supplier) and perfect competition with many suppliers. It has the following properties:

- More than one producing firm
- All firms produce one homogeneous product (no product differentiation)
- No cooperation among firms (no collusion)
- Firms have **market power** - each firm's output decision affects the good's price
- Fixed number of firms
- Firms compete in quantities, and choose them simultaneously
- Economically rational and strategically acting firms, seeking to maximize profit given their competitors' decisions

Cournot duopoly

Two identical firms produce the same good in quantities Q_1, Q_2 with linear supply functions and the same marginal cost:

$$C_1(Q_1) = c \cdot Q_1$$
$$C_2(Q_2) = c \cdot Q_2$$

The price is give through a linear demand function (with $a > c$):

$$p(Q_1, Q_2) = p(Q_1 + Q_2) = a - b \cdot (Q_1 + Q_2)$$

The profit functions of the two firms are given by:

$$\Pi_1(Q_1) = p(Q_1 + Q_2) \cdot Q_1 - C_1(Q_1)$$
$$\Pi_2(Q_2) = p(Q_1 + Q_2) \cdot Q_2 - C_2(Q_2)$$

Consider the maximisation of the profit Π_1 of firm 1 assuming that the production Q_2 of firm 2 is constant and not affected by Q_1:

$$\frac{d\Pi_1}{dQ_1} = p(Q_1 + Q_2) + \frac{dp}{dQ_1} \cdot Q_1 - \frac{dC_1}{dQ_1} = a - b \cdot (Q_1 + Q_2) - bQ_1 - c$$
$$= a - c - 2bQ_1 - bQ_2 = 0$$

Similarly for the maximisation of the profit Π_2 of firm 2:

$$\frac{d\Pi_2}{dQ_2} = p(Q_1 + Q_2) + \frac{dp}{dQ_2} \cdot Q_2 - \frac{dC_2}{dQ_2} = a - b \cdot (Q_1 + Q_2) - bQ_2 - c$$
$$= a - c - bQ_1 - 2bQ_2 = 0$$

From the second equation we can substitute $bQ_1 = a - c - 2bQ_2$ into the first to get:

$$Q_2^* = \frac{a - c}{3b} = Q_1^*$$

(note the symmetry between the identical firms) and as a result $p^* = \frac{a+2c}{3}$.

Interpolation from monopoly to polipoly

Using the Cournot model we can interpolate between a monopoly $N = 1$ and perfect competition for the case of linear supply costs $C_i(Q_i) = c \cdot Q_i$ and linear demand function $p_D(\sum_i Q) = a - b \cdot \sum_i Q_i$:

market type	N	price	sales
monopoly	1	$p = \frac{a+c}{2}$	$Q = \frac{a-c}{2b}$
duopoly	2	$p = \frac{a+2c}{3}$	$Q_i = \frac{a-c}{3b}$
oligopoly	N	$p = \frac{a+Nc}{N+1}$	$Q_i = \frac{a-c}{b(N+1)}$
polipoly	$N \to \infty$	$p = c$	$\sum_i Q_i = \frac{a-c}{b}$