Complex Renewable Energy Networks
Summer Semester 2017, Lecture 7

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Optimisation Revision

Optimisation problem

We have an objective function $f : \mathbb{R}^k \to \mathbb{R}$

$$\max_x f(x)$$

$[x = (x_1, \dots x_k)]$ subject to some constraints within $\mathbb{R}^k$:

$$\begin{array}{cccc} g_i(x) = c_i & \leftrightarrow & \lambda_i & i = 1, \dots n \\ h_j(x) \leq d_j & \leftrightarrow & \mu_j & j = 1, \dots m \end{array}$$

λ_i and μ_j are the KKT multipliers (basically Lagrange multipliers) we introduce for each constraint equation; it measures the change in the objective value of the optimal solution obtained by relaxing the constraint (shadow price).

KKT conditions

The Karush-Kuhn-Tucker (KKT) conditions are necessary conditions that an optimal solution x^*, μ^*, λ^* always satisfies (up to some regularity conditions):

1. Stationarity: For $l = 1, \ldots k$
$$\frac{\partial \mathcal{L}}{\partial x_l} = \frac{\partial f}{\partial x_l} - \sum_i \lambda_i^* \frac{\partial g_i}{\partial x_l} - \sum_j \mu_j^* \frac{\partial h_j}{\partial x_l} = 0$$
2. Primal feasibility:
$$g_i(x^*) = c_i$$
$$h_j(x^*) \leq d_j$$
3. Dual feasibility: $\mu_j^* \geq 0$
4. Complementary slackness: $\mu_j^*(h_j(x^*) - d_j) = 0$

min/max and signs

If the problem is a maximisation problem (like above), then $\mu_j^* \geq 0$ since $\mu_j = \frac{\partial \mathcal{L}}{\partial d_j}$ and if we increase d_j in the constraint $h_j(x) \leq d_j$, then the feasible space can only get bigger. Since if $X \subseteq X'$

$$\max_{x \in X} f(x) \leq \max_{x \in X'} f(x)$$

then the objective value at the optimum point can only get bigger, and thus $\mu_j^* \geq 0$. (If $d_j \to \infty$ then the constraint is no longer binding, if $d_j \to -\infty$ then the feasible space vanishes.)

If however the problem is a minimisation problem (e.g. cost minimisation) then we can use

$$\min_{x \in X} f(x) = -\max_{x \in X} [-f(x)]$$

We can keep our definition of the Lagrangian and almost all the KKT conditions, but we have a change of sign $\mu_j^* \leq 0$, since

$$\min_{x \in X} f(x) \geq \min_{x \in X'} f(x)$$

The λ_i^* also change sign.

Electricity Markets from Perspective of Single Generators and Consumers

Efficient Markets for the short-run

Assume investments already made in generators and and consumption assets (factories, machines, etc.).

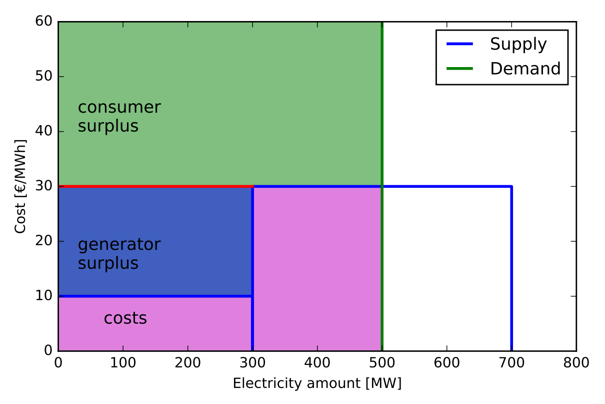
Assume all actors are price takers (i.e. nobody can exercise market power) and we have perfect competition.

How do we allocate production and consumption in the most efficient way?

I.e. we are interested in the short-run “static” efficiency.

Last time: 2 generators at a node

Last time we saw an example with 2 generators at a single node:



We're now going to look at the economics theory behind this notion of surplus.

Generator Cost Function

A generator has a cost or supply function $C(Q)$ in €/h, which gives the total operating costs (of fuel, etc.) for a given rate of electricity generation Q MW.

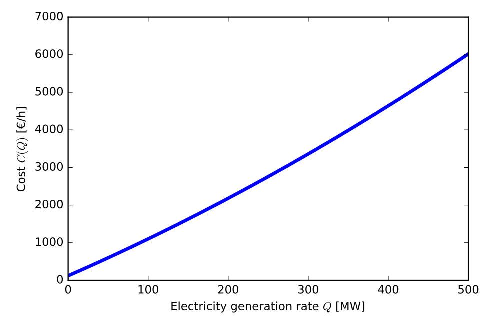
This is the integral of the purple area in the previous slide.

Typically the generator has a higher cost for a higher rate of generation Q, i.e. the first derivative is positive $C'(Q) > 0$. For most generators the rate at which cost increases with rate of production itself increases as the rate of production increases, i.e. $C''(Q) > 0$.

Cost Function: Example

A gas generator has a cost function which depends on the rate of electricity generation Q [€/h] according to

$$C(Q) = 0.005\ Q^2 + 9.3\ Q + 120$$



Note that the slope is always positive and becomes more positive for increasing Q. The curve does not start at the origin because of startup costs, no load costs, etc.

Optimal generator behaviour

We assume that the generator is a price-taker, i.e. they cannot influence the price by changing the amount they generate.

Suppose the market price is λ €/MWh. For a generation rate Q, the revenue is λQ and the generator should adjust their generation rate Q to maximise their net generation surplus, i.e. their profit:

$$\max_Q [\lambda Q - C(Q)]$$

This optimisation problem is optimised for $Q = Q^*$ where

$$C'(Q^*) \equiv \frac{dC}{dQ}(Q^*) = \lambda$$

[Check units: $\frac{dC}{dQ}$ has units $\frac{\text{€/h}}{\text{MW}} = $ €/MWh.]

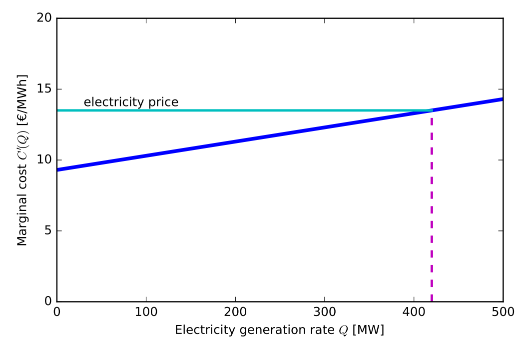
I.e. the generator increases their output until they make a net loss for any increase of generation.

$C'(Q)$ is known as the marginal cost curve, which shows, for each rate of generation Q what price λ the generator should be willing to supply at.

Marginal cost function: Example

For our example the marginal function is given by

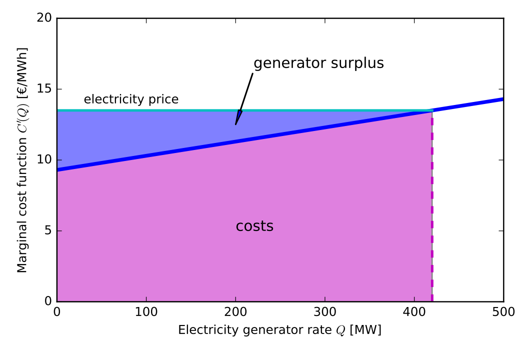
$$C'(Q) = 0.001\ Q + 9.3$$



Generator surplus

The area under the curve is generator costs, which as the integral of a derivative, just gives the cost function $C(Q)$ again, up to a constant.

The generator surplus is the profit the generator makes by having costs below the electricity price.

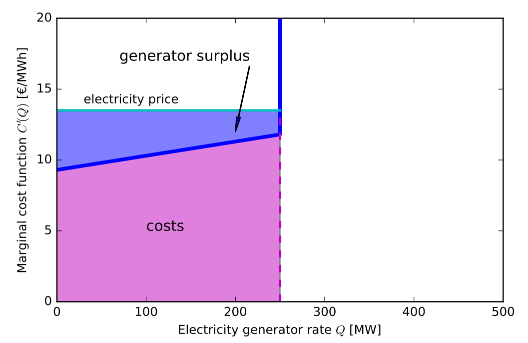


Limits to generation

Note that it is quite common for generators to be limited by e.g. their capacity, which may become a binding, i.e. limiting factor before the price plays a role, e.g.

$$Q \leq Q^{\max}$$

In the following case the optimal generation is at $Q^* = Q^{\max} =$ 250 MW. We have a binding constraint and can define a shadow price μ, which indicates the benefit of relaxing the constraint $\mu^* = \lambda - C'(Q^*)$.



Consumer behaviour: Theory

Suppose for some given period a consumer consumes electricity at a rate of Q MW.

Their utility or value function $U(Q)$ in €/h is a measure of their benefit for a given consumption rate Q.

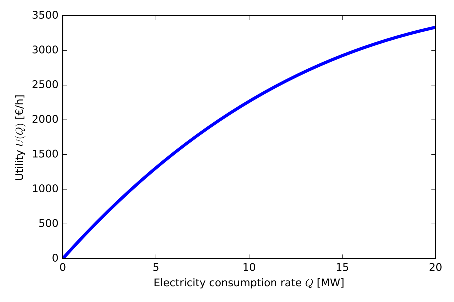
For a firm this could be the profit related to this electricity consumption from manufacturing goods.

Typical the consumer has a higher utility for higher Q, i.e. the first derivative is positive $U'(Q) > 0$. By assumption, the rate of value increase with consumption decreases the higher the rate of consumption, i.e. $U''(Q) < 0$.

Utility: Example

A widget manufacturer has a utility function which depends on the rate of electricity consumption Q [€/h] as

$$U(Q) = 0.0667\ Q^3 - 8\ Q^2 + 300\ Q$$



Note that the slope is always positive, but becomes less positive for increasing Q.

Optimal consumer behaviour

We assume that the consumer is a price-taker, i.e. they cannot influence the price by changing the amount they consume.

Suppose the market price is λ €/MWh. The consumer should adjust their consumption rate Q to maximise their net surplus

$$\max_{Q} \left[U(Q) - \lambda Q \right]$$

This optimisation problem is optimised for $Q = Q^*$ where

$$U'(Q^*) \equiv \frac{dU}{dQ}(Q^*) = \lambda$$

[Check units: $\frac{dU}{dQ}$ has units $\frac{\text{€/h}}{\text{MW}} = \text{€/MWh}$.]

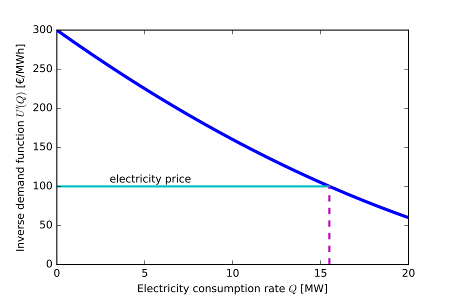
I.e. the consumer increases their consumption until they make a net loss for any increase of consumption.

$U'(Q)$ is known as the inverse demand curve or marginal utility curve, which shows, for each rate of consumption Q what price λ the consumer should be willing to pay.

Inverse demand function: Example

For our example the inverse demand function is given by

$$U'(Q) = 0.2\ Q^2 - 16\ Q + 300$$



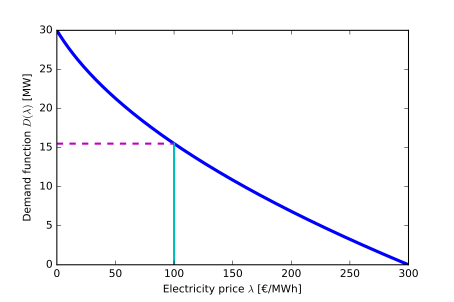
It's called the *inverse* demand function, because the demand function is the function you get from reversing the axes.

Inverse demand function: Example

The demand function $D(\lambda)$ gives the demand Q as a function of the price λ. $D(U'(Q)) = Q$.

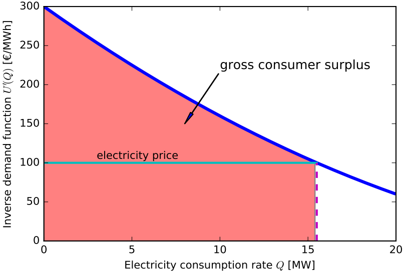
For our example the demand function is given by

$$D(\lambda) = -((\lambda + 20)/0.2)^{0.5} + 40$$



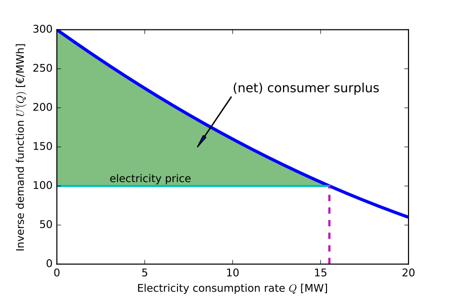
Gross consumer surplus

The area under the inverse demand curve is the gross consumer surplus, which as the integral of a derivative, just gives the utility function $U(Q)$ again, up to a constant.



Net consumer surplus

The more relevant net consumer surplus, or just consumer surplus is the net gain the consumer makes by having utility above the electricity price.

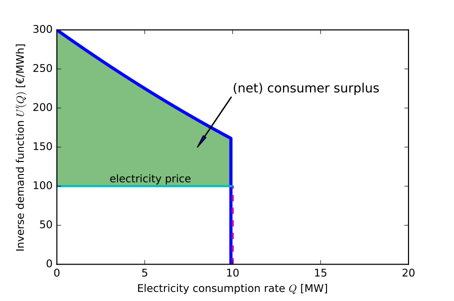


Limits to consumption

Note that it is quite common for consumption to be limited by other factors before the electricity price becomes too expensive, e.g. due to the size of electrical machinery. This gives an upper bound

$$Q \leq Q^{\max}$$

In the following case the optimal consumption is at $Q^* = Q^{\max} =$ 10 MW. We have a binding constraint and can define a shadow price μ, which indicates the benefit of relaxing the constraint $\mu^* = U'(Q^*) - \lambda$.

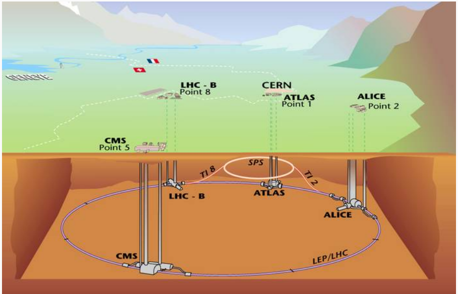


Consumers can delay their consumption

Besides changing the amount of electricity consumption, consumers can also shift their consumption in time.

For example electric storage heaters use cheap electricity at night to generate heat and then store it for daytime.

The LHC particle accelerator does not run in the winter, when prices are higher (see `http://home.cern/about/engineering/powering-cern`). Summer demand: 200 MW, corresponds to a third of Geneva, equal to peak demand of Rwanda (!); winter only 80 MW.



Source: CERN

Consumers can also move location

Aluminium smelting is an electricity-intensive process. Aluminium smelters will often move to locations with cheap and stable electricity supplies, such as countries with lots of hydroelectric power. For example, 73% of Iceland's total power consumption in 2010 came from aluminium smelting.

Aluminium costs around US$ 1500/ tonne to produce.

Electricity consumption: 15 MWh/tonne.

At Germany consumer price of €300 / MWh, this is €4500 / tonne. Uh-oh!!!

If electricity is 50% of cost, then need $750/tonne to go on electricity $\Rightarrow$ 750/15 \$/MWh = 50 \$/MWh.

Summary: Consumers and Generators

Generators: A generator has a cost or supply function $C(Q)$ in €/h, which gives the costs (of fuel, etc.) for a given rate of electricity generation Q MW. If the market price is λ €/MWh, the revenue is λQ and the generator should adjust their generation rate Q to maximise their net generation surplus, i.e. their profit:

$$\max_Q \left[\lambda Q - C(Q)\right]$$

Consumers: Their utility or value function $U(Q)$ in €/h is a measure of their benefit for a given consumption rate Q. For a given price λ they adjust their consumption rate Q such that their net surplus is maximised:

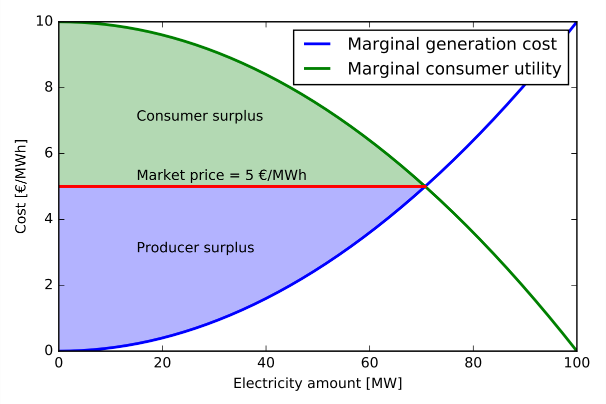
$$\max_Q \left[U(Q) - \lambda Q\right]$$

Supply and Demand at a Single Node

Setting the quantity and price

Total welfare (consumer and generator surplus) is maximised if the total quantity is set where the marginal cost and marginal utility curves meet.

If the price is also set from this point, then the individual optimal actions of each actor will achieve this result in a perfect decentralised market.



The result of optimisation

This is the result of maximising the total economic welfare, the sum of the consumer and the producer surplus for consumers with consumption Q_i^B and generators generating with rate Q_i^S:

$$\max_{\{Q_i^B\},\{Q_i^S\}} \left[\sum_i U_i(Q_i^B) - \sum_i C_i(Q_i^S) \right]$$

subject to the supply equalling the demand in the balance constraint:

$$\sum_i Q_i^B - \sum_i Q_i^S = 0 \qquad \leftrightarrow \qquad \lambda$$

and any other constraints (e.g. limits on generator capacity, etc.).

Market price λ is the shadow price of the balance constraint, i.e. the cost of supply an extra increment 1 MW of demand.

Why decentralised markets work (in theory)

We will now show our main result:

Welfare-maximisation through decentralised markets

The welfare-maximising combination of production and consumption can be achieved by the decentralised profit-maximising decisions of producers and the utility-maximising decisions of consumers, provided that:

- The market price is equal to the constraint marginal value of the overall supply-balance constraint in the welfare maximisation problem
- All producers and consumers are price-takers

KKT and Welfare Maximisation 1/2

Apply KKT now to maximisation of total economic welfare:

$$\max_{\{Q_i^B\},\{Q_i^S\}} f(\{Q_i^B\},\{Q_i^S\}) = \left[\sum_i U_i(Q_i^B) - \sum_i C_i(Q_i^S)\right]$$

subject to the balance constraint:

$$g(\{Q_i^B\},\{Q_i^S\}) = \sum_i Q_i^B - \sum_i Q_i^S = 0 \qquad \leftrightarrow \qquad \lambda$$

and any other constraints (e.g. limits on generator capacity, etc.).

Our optimisation variables are $\{x\} = \{Q_i^B\} \cup \{Q_i^S\}$.

We get from stationarity:

$$0 = \frac{\partial f}{\partial Q_i^B} - \sum_i \lambda^* \frac{\partial g}{\partial Q_i^B} = U_i'(Q_i^B) - \lambda^* = 0$$

$$0 = \frac{\partial f}{\partial Q_i^S} - \sum_i \lambda^* \frac{\partial g}{\partial Q_i^S} = -C_i'(Q_i^S) + \lambda^* = 0$$

KKT and Welfare Maximisation 2/2

So at the optimal point of maximal total economic welfare we get the same result as if everyone maximises their own welfare separately:

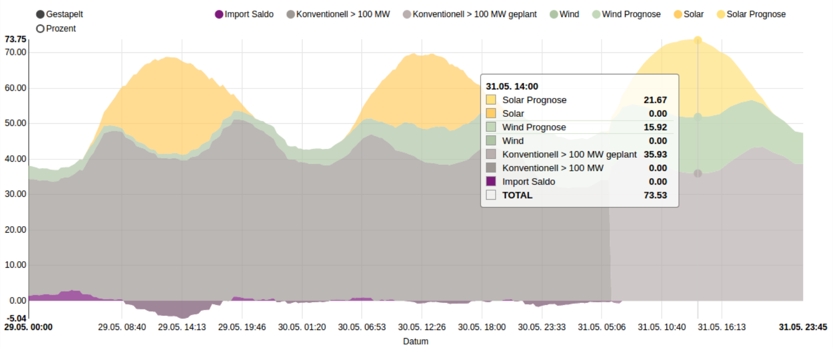
$$U_i'(Q_i^B) = \lambda^*$$
$$C_i'(Q_i^S) = \lambda^*$$

This is the CENTRAL result of microeconomics.

If we have further inequality constraints that are binding, then these equations will receive additions with $\mu_i^* > 0$.

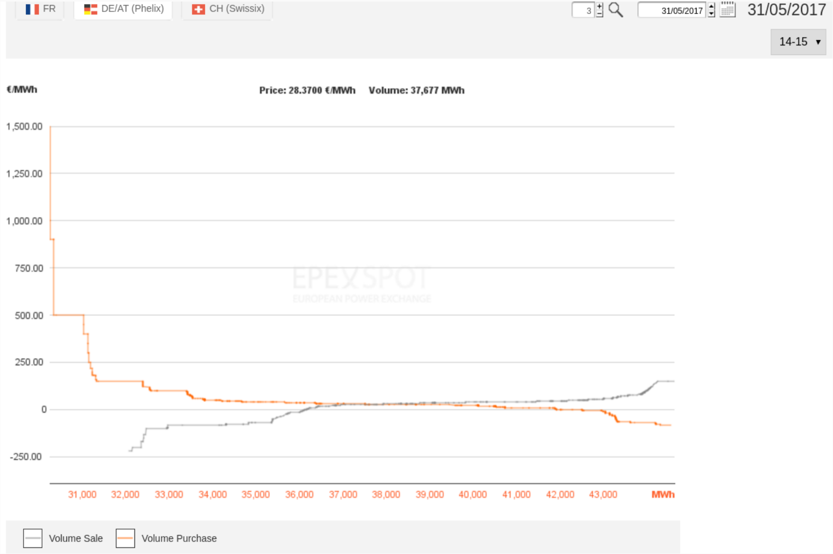
Power Production Right Now

Here's the forecast of load, wind, solar and conventional generation right now in Germany (link):

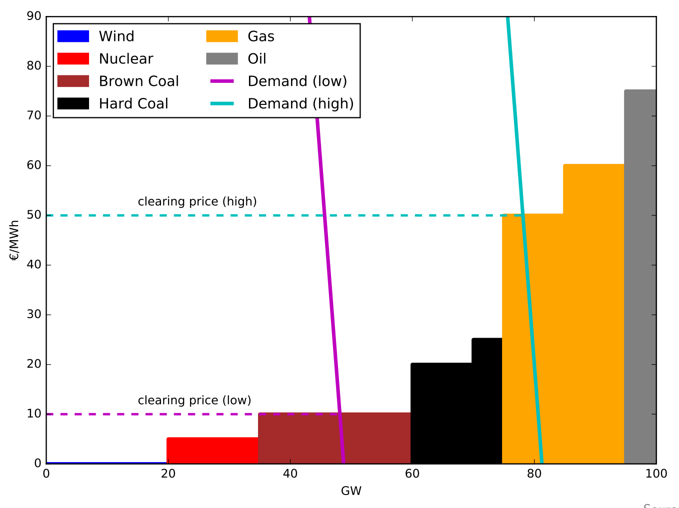


Supply-Demand Curve Right Now

Here's the supply-demand curve for Germany-Austria right now (link)

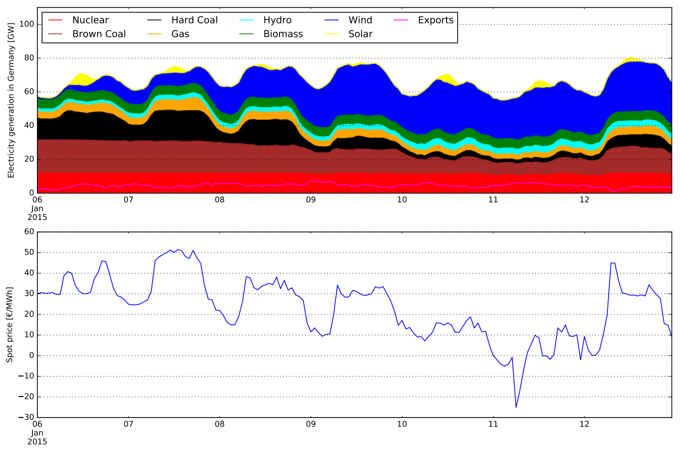


Effect of varying demand for fixed generation

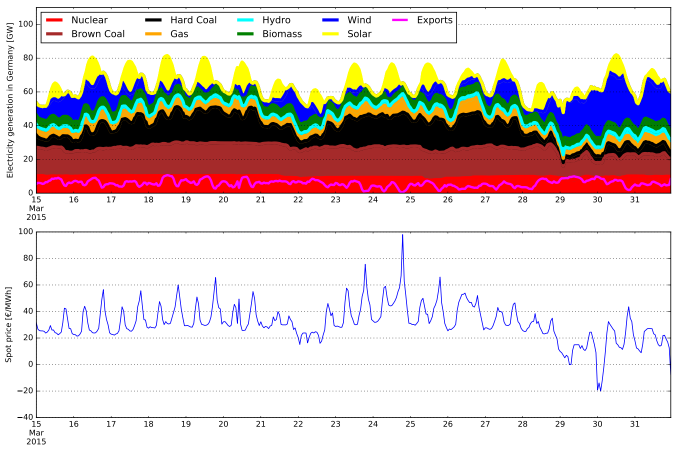


Source: Tom Brown

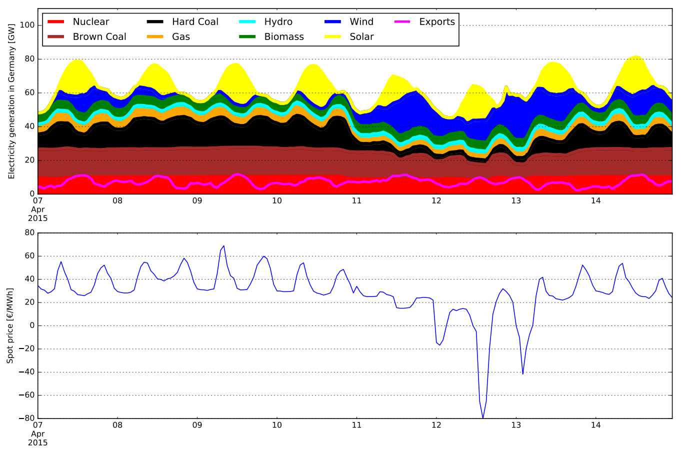
Example market 1/3



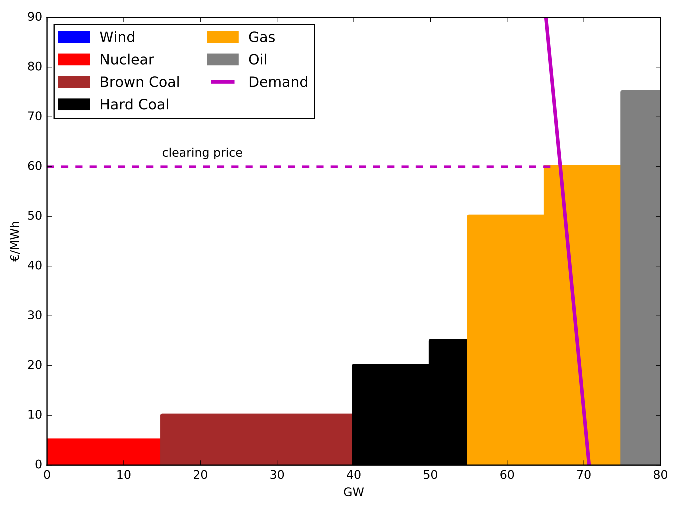
Example market 2/3



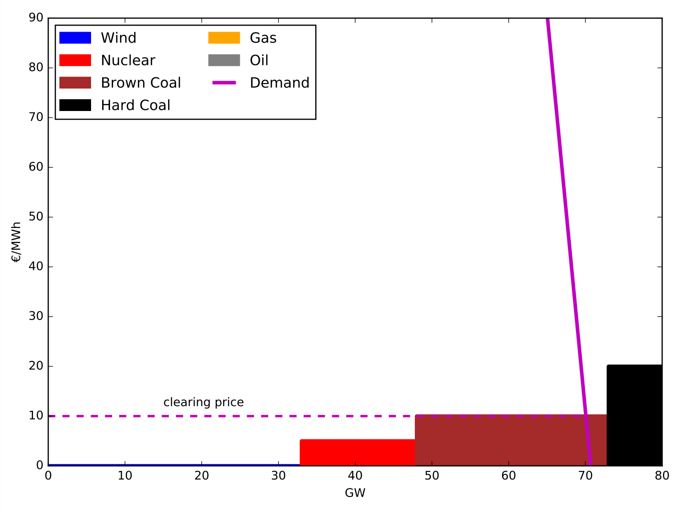
Example market 3/3



Effect of varying renewables: fixed demand, no wind

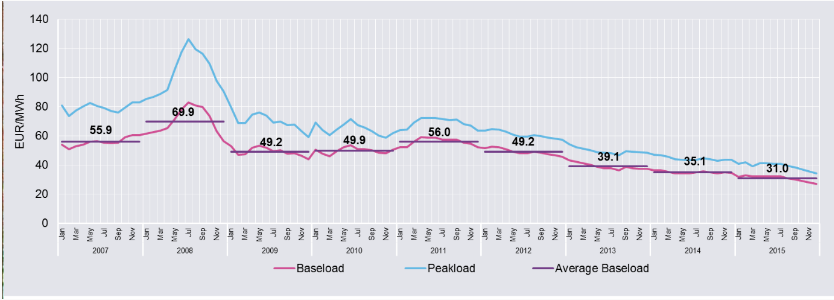


Effect of varying renewables: fixed demand, 35 GW wind



Spot market price development

As a result of so much zero-marginal-cost renewable feed-in, spot market prices have been steadily decreasing:



Source: Agora Energiewende

Merit Order Effect

To summarise:

- Renewables have zero marginal cost
- As a result they enter at the bottom of the merit order, reducing the price at which the market clears
- This pushes non-CHP gas and hard coal out of the market
- This is unfortunate, because among the fossil fuels, gas and hard coal are the most flexible and produce the *lowest* CO_2 per MWh
- It also massively reduces the profits that nuclear and brown coal make
- Will there be enough backup power plants for times with no wind/solar?

This has led to lots of political tension...

Optimisation Energy System Operation: Network

Simplify representation of consumers and generators

We will now return to the simplified world of last lecture, where all the generator cost functions are linear

$$C_s(g_s) = o_s g_s$$

and each generator has limited output $0 \leq g_s \leq G_s$.

[The variable g_s indexed by s is equivalent to Q_i^S labelled by i above.]

We also fix the demand so that it is not subject to optimisation. This is equivalent to having a single consumer with very high marginal utility $V >> o_s \; \forall s$ and $d < \sum_s G_s$ for an inelastic demand level d

$$U(Q^B) = VQ^B \text{ for } Q^B \leq d$$
$$U(Q^B) = 0 \text{ for } Q^B > d$$

Simplify representation of consumers and generators

In this case we get for our welfare maximisation:

$$\max_{Q^B,\{g_s\}} \left[VQ^B - \sum_s o_s g_s \right]$$

subject to:

$$\begin{aligned} Q^B - \sum_s g_s &= 0 && \leftrightarrow && \lambda \\ Q^B &\leq d \\ g_s &\leq G_s && \leftrightarrow && \bar{\mu}_s \\ -g_s &\leq 0 && \leftrightarrow && \underline{\mu}_s \end{aligned}$$

Since $V >> o_s$ we will always get $Q^{B*} = d$ and we can drop this from the optimisation.

Simplify representation of consumers and generators

We thus get:

$$\max_{\{g_s\}} \left[Vd - \sum_s o_s g_s \right]$$

subject to:

$$\begin{aligned} d - \sum_s g_s = 0 \qquad &\leftrightarrow \qquad \lambda \\ g_s \leq G_s \qquad &\leftrightarrow \qquad \bar{\mu}_s \\ -g_s \leq 0 \qquad &\leftrightarrow \qquad \underline{\mu}_s \end{aligned}$$

Simplify representation of consumers and generators

Finally we drop the constant utility Vd from the objective function (it has no influence on the results) and use

$$\min_{x\in X} f(x) = -\max_{x\in X}[-f(x)]$$

to turn the maximisation of total welfare into a cost minimisation problem:

$$\min_{\{g_s\}} \sum_s o_s g_s$$

such that:

$$\sum_s g_s - d = 0 \qquad \leftrightarrow \qquad \lambda$$

$$g_s \leq G_s \qquad \leftrightarrow \qquad \bar{\mu}_s$$

$$-g_s \leq 0 \qquad \leftrightarrow \qquad \underline{\mu}_s$$

NB: Because the signs of the KKT multipliers change when we go from maximisation to minimisation, we've also changed the sign of the balance constraint to keep the marginal price λ positive.

Several generators at different nodes in a network

Now let's suppose we have several nodes i with different loads and different generators, with flows f_ℓ in the network lines.

Now we have additional optimisation variables f_ℓ AND additional constraints:

$$\min_{\{g_{i,s}\},\{f_\ell\}} \sum_{i,s} o_{i,s} g_{i,s}$$

such that demand is met either by generation or by the network at each node i

$$\sum_s g_{i,s} - d_i = \sum_\ell K_{i\ell} f_\ell \qquad \leftrightarrow \qquad \lambda_i$$

and generator constraints are satisified

$$g_{i,s} \leq G_{i,s} \qquad \leftrightarrow \qquad \bar{\mu}_{i,s}$$
$$-g_{i,s} \leq 0 \qquad \leftrightarrow \qquad \underline{\mu}_{i,s}$$

Several generators at different nodes in a network

In addition we have constraints on the line flows.

First, they have to satisfy Kirchoff's Voltage Law around each closed cycle c:

$$\sum_c C_{\ell c} x_\ell f_\ell = 0 \qquad \leftrightarrow \qquad \lambda_c$$

and in addition the flows cannot overload the thermal limits, $|f_\ell| \leq F_\ell$

$$\begin{aligned} f_\ell \leq F_\ell &\qquad \leftrightarrow \qquad \bar{\mu}_\ell \\ -f_\ell \leq -F_\ell &\qquad \leftrightarrow \qquad \underline{\mu}_\ell \end{aligned}$$

Simplest example: two nodes connected by a single line

At node 1 we have demand of $d_1 = 100$ MW and a generator with costs $o_1 = 10$ €/MWh and a capacity of $G_1 = 300$ MW.

At node 2 we have demand of $d_2 = 100$ MW and a generator with costs $o_1 = 20$ €/MWh and a capacity of $G_2 = 300$ MW.

What happens if the capacity of the line connecting them is $F_\ell = 0$?

What about $F_\ell = 50$ MW?

What about $F_\ell = \infty$?

Storage Optimisation

Storage equations

Now, like the network case where we add different nodes i with different loads, for storage we have to consider different time periods t.

Label conventional generators by s, storage by r and now minimise

$$\min_{\{g_{i,s,t}\},\{g_{i,r,t,\text{store}}\},\{g_{i,r,t,\text{dispatch}}\},\{f_{\ell,t}\}} \left[\sum_{i,s,t} o_{i,s} g_{i,s,t} + \sum_{i,r,t} o_{i,r,\text{store}}\, g_{i,r,t,\text{store}} + \sum_{i,r,t} o_{i,r,\text{dispatch}}\, g_{i,r,t,\text{dispatch}}\right]$$

The power balance constraints are now (cf. Lecture 4) for each node i and time t that the demand is met either by generation, storage or network flows:

$$\sum_s g_{i,s,t} + \sum_r (g_{i,r,t,\text{dispatch}} - g_{i,r,t,\text{store}}) - d_{i,t} = \sum_\ell K_{i\ell} f_{\ell,t} \quad \leftrightarrow \quad \lambda_{i,t}$$

Storage equations

We have constraints on normal generators

$$0 \leq g_{i,s,t} \leq G_{i,s}$$

and on the storage

$$0 \leq g_{i,r,t,\text{dispatch}} \leq G_{i,r,\text{dispatch}}$$
$$0 \leq g_{i,r,t,\text{store}} \leq G_{i,r,\text{store}}$$

The energy level of the storage is given by

$$e_{i,r,t} = \eta_0 e_{i,r,t-1} + \eta_1 g_{i,r,t,\text{store}} - \eta_2^{-1} g_{i,r,t,\text{dispatch}}$$

and limited by

$$0 \leq e_{i,r,t} \leq E_{i,r}$$

Storage equations

Finally for the flows we repeat the constraints for each time t.

We have KVL for each cycle c and time t

$$\sum_c C_{\ell c} x_\ell f_{\ell,t} = 0 \qquad \leftrightarrow \qquad \lambda_{c,t}$$

and in addition the flows cannot overload the thermal limits, $|f_{\ell,t}| \leq F_\ell$

$$\begin{aligned} f_{\ell,t} &\leq F_\ell & \qquad \leftrightarrow \qquad & \bar{\mu}_{\ell,t} \\ -f_{\ell,t} &\leq -F_\ell & \qquad \leftrightarrow \qquad & \underline{\mu}_{\ell,t} \end{aligned}$$

Preview: Investment optimisation

Preview for next time:

Next time we will also optimise investment in the capacities of generators, storage and network lines, to maximise long-run efficiency.

We will promote the capacities $G_{i,s}$, $G_{i,r,*}$, $E_{i,r}$ and F_ℓ to optimisation variables.