Complex Renewable Energy Networks
Summer Semester 2017, Lecture 6

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Optimisation: Motivation

What to do about variable renewables?

Backup energy costs money and may also cause CO_2 emissions.

Curtailing renewable energy is also a waste.

We have discussed the first 3 of the 4 solutions suggested in the first lecture:

1. Smoothing stochastic variations of renewable feed-in over larger areas, e.g. the whole of European continent.
2. Using storage to shift energy from times of surplus to deficit.
3. Shifting demand to different times, when renewables are abundant.
4. Consuming the electricity in other sectors, e.g. transport or heating.

Before tackling sector-coupling, we will take a few lectures to discuss optimisation in energy networks as a tool to assess these options.

Why optimisation?

In the energy system we have lots of degrees of freedom:

1. Power plant and storage dispatch
2. Renewables curtailment
3. Dispatch of network elements (e.g. High Voltage Direct Current (HVDC) lines)
4. Capacities of everything when considering investment

but we also have to respect physical constraints:

1. Meet energy demand
2. Do not overload generators or storage
3. Do not overload network

and we want to do this while minimising costs. Solution: optimisation.

Optimisation: Introduction

A simple optimisation problem

Consider the following problem. We have a function $f(x, y)$ of two variables $x, y \in \mathbb{R}$

$$f(x, y) = 3x$$

and we want to find the maximum of this function in the $x - y$ plane

$$\max_{x,y \in \mathbb{R}} f(x, y)$$

subject to the following constraints

$$x + y \leq 4 \tag{1}$$

$$x \geq 0 \tag{2}$$

$$y \geq 1 \tag{3}$$

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subject to the following constraints

$$x + y \leq 4 \quad (1)$$

$$x \geq 0 \quad (2)$$

$$y \geq 1 \quad (3)$$

Optimal solution: $x^* = 3, y^* = 1, f(x^*, y^*) = 9$.

Another simple optimisation problem

We can also have equality constraints. Consider the maximum of this function in the $x - y - z$ space

$$\max_{x,y,z \in \mathbb{R}} f(x, y, z) = (3x + 5z)$$

subject to the following constraints

$$\begin{aligned} x + y &\leq 4 \\ x &\geq 0 \\ y &\geq 1 \\ z &= 2 \end{aligned}$$

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Optimal solution: $x^* = 3, y^* = 1, z^* = 2, f(x^*, y^*, z^*) = 19.$

Energy system mapping to an optimisation problem

This optimisation problem has the same basic form as our energy system considerations:

Objective function to minimise	$\leftrightarrow$	Minimise total costs
Optimisation variables	$\leftrightarrow$	Physical degrees of freedom (power plant dispatch, etc.)
Constraints	$\leftrightarrow$	Physical constraints (overloading, etc.)

Before we apply optimisation to the energy system, we'll do some theory.

Optimisation: Theory

Optimisation problem

We have an objective function $f : \mathbb{R}^k \to \mathbb{R}$

$$\max_x f(x)$$

$[x = (x_1, \ldots x_k)]$ subject to some constraints within $\mathbb{R}^k$:

$$\begin{array}{cccc} g_i(x) = c_i & \leftrightarrow & \lambda_i & i = 1, \ldots n \\ h_j(x) \leq d_j & \leftrightarrow & \mu_j & j = 1, \ldots m \end{array}$$

λ_i and μ_j are the KKT multipliers (basically Lagrange multipliers) we introduce for each constraint equation; it measures the change in the objective value of the optimal solution obtained by relaxing the constraint (shadow price).

Feasibility

The space $X \subset \mathbb{R}^k$ which satisfies

$$\begin{array}{cccc} g_i(x) = c_i & \leftrightarrow & \lambda_i & i = 1, \dots n \\ h_j(x) \leq d_j & \leftrightarrow & \mu_j & j = 1, \dots m \end{array}$$

is called the feasible space.

It will have dimension lower than k if there are independent equality constraints.

It may also be empty (e.g. $x \geq 1, x \leq 0$ in $\mathbb{R}$), in which case the optimisation problem is called infeasible.

It can be convex or non-convex.

If all the constraints are affine, then the feasible space is a convex polygon.

Lagrangian

We now study the Lagrangian function

$$\mathcal{L}(x, \lambda, \mu) = f(x) - \sum_i \lambda_i \left[g_i(x) - c_i\right] - \sum_j \mu_j \left[h_j(x) - d_j\right]$$

We've built this function using the variables λ_i and μ_j to better understand the optimal solution of $f(x)$ given the constraints.

The optima of $\mathcal{L}(x, \lambda, \mu)$ tell us important information about the optima of $f(x)$ given the constraints.

It is entirely analogous to the physics Lagrangian $L(x, \dot{x}, \lambda)$ except we have no explicit time dependence $\dot{x}$ and we have additional constraints which are inequalities.

We can already see that if $\frac{\partial \mathcal{L}}{\partial \lambda_i} = 0$ then the equality constraint $g_i(x) = c$ will be satisfied.

[Beware: $\pm$ signs appear differently in literature, but have been chosen here such that $\lambda_i = \frac{\partial \mathcal{L}}{\partial c_i}$ and $\mu_j = \frac{\partial \mathcal{L}}{\partial d_j}$.]

KKT conditions

The Karush-Kuhn-Tucker (KKT) conditions are necessary conditions that an optimal solution x^*, μ^*, λ^* always satisfies (up to some regularity conditions):

1. Stationarity: For $l = 1, \dots k$
$$\frac{\partial \mathcal{L}}{\partial x_l} = \frac{\partial f}{\partial x_l} - \sum_i \lambda_i^* \frac{\partial g_i}{\partial x_l} - \sum_j \mu_j^* \frac{\partial h_j}{\partial x_l} = 0$$
2. Primal feasibility:
$$g_i(x^*) = c_i$$
$$h_j(x^*) \leq d_j$$
3. Dual feasibility: $\mu_j^* \geq 0$
4. Complementary slackness: $\mu_j^*(h_j(x^*) - d_j) = 0$

Complementarity slackness for inequality constraints

We have for each inequality constraint

$$\mu_j^* \geq 0$$
$$\mu_j^*(h_j(x^*) - d_j) = 0$$

So either the inequality constraint is binding

$$h_j(x^*) = d_j$$

and we have $\mu_j^* \geq 0$.

Or the inequality constraint is NOT binding

$$h_j(x^*) < d_j$$

and we therefore MUST have $\mu_j^* = 0$.

If the inequality constraint is non-binding, we can remove it from the optimisation problem, since it has no effect on the optimal solution.

Nota Bene

1. The KKT conditions are only sufficient for optimality of the solution under certain conditions, e.g. linearity of the problem.

2. Since at the optimal solution we have $g_i(x^*) = c_i$ for equality constraints and $\mu_j^*(h_j(x^*) - d_j) = 0$, we have

$$\mathcal{L}(x^*, \lambda^*, \mu^*) = f(x^*)$$

Return to simple optimisation problem

We want to find the maximum of this function in the $x - y$ plane

$$\max_{x,y \in \mathbb{R}} f(x, y) = 3x$$

subject to the following constraints (now with KKT multipliers)

$$\begin{aligned} x + y \leq 4 \qquad &\leftrightarrow \qquad \mu_1 \\ -x \leq 0 \qquad &\leftrightarrow \qquad \mu_2 \\ -y \leq -1 \qquad &\leftrightarrow \qquad \mu_3 \end{aligned}$$

We know the optimal solution in the primal variables $x^* = 3, y^* = 1, f(x^*, y^*) = 9$.

What about the dual variables μ_i?

Since the second constraint is not binding, by complementarity $\mu_2^*(-x^* - 0) = 0$ we have $\mu_2^* = 0$. To find μ_1^* and μ_3^* we have to do more work.

Simple problem with KKT conditions

We use stationarity for the optimal point:

$$0 = \frac{\partial \mathcal{L}}{\partial x} = \frac{\partial f}{\partial x} - \sum_i \lambda_i^* \frac{\partial g_i}{\partial x} - \sum_j \mu_j^* \frac{\partial h_j}{\partial x} = 3 - \mu_1 + \mu_2$$

$$0 = \frac{\partial \mathcal{L}}{\partial y} = \frac{\partial f}{\partial y} - \sum_i \lambda_i^* \frac{\partial g_i}{\partial y} - \sum_j \mu_j^* \frac{\partial h_j}{\partial y} = -\mu_1 + \mu_3$$

From which we find:

$$\mu_1^* = 3 - \mu_2^* = 3$$

$$\mu_3^* = \mu_1^* = 3$$

Check interpretation: $\mu_j = \frac{\partial \mathcal{L}}{\partial d_j}$ with $d_j \to d_j + \varepsilon$.

Return to another simple optimisation problem

We want to find the maximum of this function in the $x - y - z$ space

$$\max_{x,y,z \in \mathbb{R}} f(x, y) = 3x + 5z$$

subject to the following constraints (now with KKT multipliers)

$$\begin{aligned} x + y \leq 4 \quad &\leftrightarrow \quad \mu_1 \\ -x \leq 0 \quad &\leftrightarrow \quad \mu_2 \\ -y \leq -1 \quad &\leftrightarrow \quad \mu_3 \\ z = 2 \quad &\leftrightarrow \quad \lambda \end{aligned}$$

We know the optimal solution in the primal variables
$x^* = 3, y^* = 1, z^* = 2, f(x^*, y^*, z^*) = 19.$

What about the dual variables μ_i, λ?

We get same solutions to $\mu_1^* = 3, \mu_2^* = 0, \mu_3^* = 3$ because they're not coupled to z direction. What about λ^*?

Another simple problem with KKT conditions

We use stationarity for the optimal point:

$$0 = \frac{\partial \mathcal{L}}{\partial z} = \frac{\partial f}{\partial z} - \sum_i \lambda_i^* \frac{\partial g_i}{\partial z} - \sum_j \mu_j^* \frac{\partial h_j}{\partial z} = 5 - \lambda^*$$

From which we find:

$$\lambda^* = 5$$

Check interpretation: $\lambda_i = \frac{\partial \mathcal{L}}{\partial c_i}$ with $c_i \to c_i + \varepsilon$.

Optimisation Energy System Operation: Single Node

Simplest example: one generator, fixed demand

These sections will follow the notation of Lecture 4.

Suppose we have a single node with demand given by d and a single conventional generator with dispatch g (our free parameter to optimise) such that the demand is met:

$$g - d = 0 \qquad \leftrightarrow \qquad \lambda$$

In addition, the dispatch g cannot be negative or overload the capacity G:

$$\begin{aligned} g \leq G \qquad &\leftrightarrow \qquad \bar{\mu} \\ -g \leq 0 \qquad &\leftrightarrow \qquad \underline{\mu} \end{aligned}$$

Suppose in addition it costs o to dispatch the generator by g (o for operating costs). We try to minimise costs, i.e.

$$\min_{g} og$$

such that the above three constraints are satisfied.

Simplest example: one generator, fixed demand

The solution is trivial. The generator dispatches to meet the demand

$$g^* = d$$

If $d > G$ then the problem is infeasible (has no solution). If the demand is non-zero then since $g^* > 0$ by complementarity we have $\underline{\mu}^* = 0$. If $d < G$ then $g^* < G$ and by complementarity we have $\bar{\mu}^* = 0$. To compute λ^* we use stationarity:

$$0 = \frac{\partial \mathcal{L}}{\partial g} = \frac{\partial f}{\partial g} - \sum_i \lambda_i^* \frac{\partial g_i}{\partial g} - \sum_j \mu_j^* \frac{\partial h_j}{\partial g} = o - \lambda^* - \bar{\mu}^* + \underline{\mu}^*$$

Thus $\lambda^* = o$, which is the cost per unit of supplying extra demand.

[If $d = G$, things get singular: $\lambda = \infty$, since there is no way to supply the extra demand.]

[There has been a subtle sign redefinition here for $\mathcal{L}$ since $\min_x f(x) = -\max_x[-f(x)]$. Sorry.]

Next simplest example: several generators, fixed demand

Suppose we have several generators with dispatch g_s and strictly ordered operating costs o_s such that $o_s < o_{s+1}$. We now minimise

$$\min_{\{g_s\}} \sum_s o_s g_s$$

such that demand is met

$$\sum_s g_s - d = 0 \qquad \leftrightarrow \qquad \lambda$$

and generator constraints are satisified

$$g_s \leq G_s \qquad \leftrightarrow \qquad \bar{\mu}_s$$
$$-g_s \leq 0 \qquad \leftrightarrow \qquad \underline{\mu}_s$$

Next simplest example: several generators, fixed demand

Stationarity gives us for each s:

$$0 = o_s - \lambda^* - \bar{\mu}_s^* + \underline{\mu}_s^*$$

and from complementarity we get

$$\bar{\mu}_s(g_s^* - G_s) = 0$$
$$\underline{\mu}_s g_s^* = 0$$

We can see by inspection that we will dispatch the cheapest generation first. Find m such that $\sum_{s=1}^{m-1} G_s < d < \sum_{s=1}^{m} G_s$.

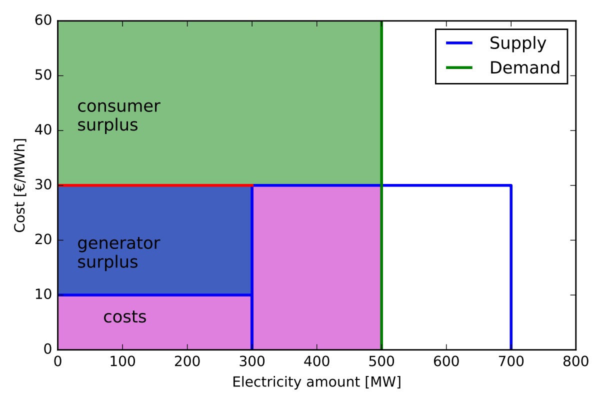
For $s \leq m-1$ we have $g_s^* = G_s$, $\underline{\mu}_s^* = 0$, $\bar{\mu}_s^* = o_s - \lambda^*$.

For $s = m$ we have $g_m^* = d - \sum_{s=1}^{m-1} G_s$ to cover what's left of the demand. Since $0 < g_m^* < G_m$ we have $\underline{\mu}_m^* = \bar{\mu}_m^* = 0$ and therefore $\lambda^* = o_m$.

Next simplest example: several generators, fixed demand

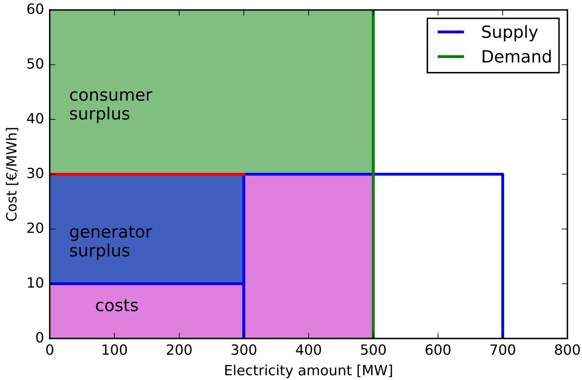
Specific example of two generators with $G_1 = 300$ MW, $G_2 = 400$ MW, $o_1 = 10$ €/MWh, $o_2 = 30$ €/MWh and $d = 500$ MW.

In this case $m = 2$, $g_1^* = G_1 = 300$ MW, $g_2^* = d - G_1 = 200$ MW, $\lambda^* = o_2$, $\underline{\mu}_i = 0$, $\bar{\mu}_2 = 0$ and $\bar{\mu}_1 = o_1 - o_2$.



Next simplest example: several generators, fixed demand

If λ^* sets the market price, then the profits that generators make by having costs below the market price is called the generator surplus. The fact that consumers are not price-responsive means that they are willing to pay a lot for their electricity; the 'profit' they make by paying less than they are prepared to is called consumer surplus. Together, these form social welfare, which economists like to maximise.



Optimisation Energy System Operation: Network

Several generators at different nodes in a network

Now let's suppose we have several nodes i with different loads and different generators, with flows f_ℓ in the network lines.

Now we have additional optimisation variables f_ℓ AND additional constraints:

$$\min_{\{g_{i,s}\},\{f_\ell\}} \sum_{i,s} o_{i,s} g_{i,s}$$

such that demand is met either by generation or by the network

$$\sum_s g_{i,s} - d_i = \sum_\ell K_{i\ell} f_\ell \qquad \leftrightarrow \qquad \lambda_i$$

and generator constraints are satisified

$$\begin{aligned} g_{i,s} &\leq G_{i,s} \qquad \leftrightarrow \qquad \bar{\mu}_{i,s} \\ -g_{i,s} &\leq 0 \qquad \leftrightarrow \qquad \underline{\mu}_{i,s} \end{aligned}$$

Several generators at different nodes in a network

In addition we have constraints on the line flows.

First, they have to satisfy Kirchoff's Voltage Law around each closed cycle c:

$$\sum_c C_{\ell c} x_\ell f_\ell = 0 \qquad \leftrightarrow \qquad \lambda_c$$

and in addition the flows cannot overload the thermal limits, $|f_\ell| \leq F_\ell$

$$\begin{aligned} f_\ell \leq F_\ell \qquad &\leftrightarrow \qquad \bar{\mu}_\ell \\ -f_\ell \leq -F_\ell \qquad &\leftrightarrow \qquad \underline{\mu}_\ell \end{aligned}$$

Simplest example: two nodes connected by a single line

At node 1 we have demand of $d_1 = 100$ MW and a generator with costs $o_1 = 10$ €/MWh and a capacity of $G_1 = 300$ MW.

At node 2 we have demand of $d_2 = 100$ MW and a generator with costs $o_1 = 20$ €/MWh and a capacity of $G_2 = 300$ MW.

What happens if the capacity of the line connecting them is $F_\ell = 0$?

What about $F_\ell = 50$ MW?

What about $F_\ell = \infty$?

Next time: Storage and capacity optimisation

Given a desired CO_2 reduction, what is the most cost-effective system?

$$\text{Min}\begin{pmatrix}\text{Yearly system}\\ \text{costs}\end{pmatrix} = \sum_{n}\begin{pmatrix}\text{Annualised}\\ \text{capital costs}\end{pmatrix} + \sum_{n,t}(\text{Marginal costs})$$

subject to

- meeting energy demand at each node n (e.g. countries) and time t (e.g. hours of year)
- wind, solar, hydro (variable renewables) availability $\forall n, t$
- electricity/gas transmission constraints between nodes
- (installed capacity) $\leq$ (geographical potential)
- CO_2 constraint / RE share covering demand
- Constraints on total volume of transmission lines