3.36pt

Complex Renewable Energy Networks
Summer Semester 2017, Lecture 3

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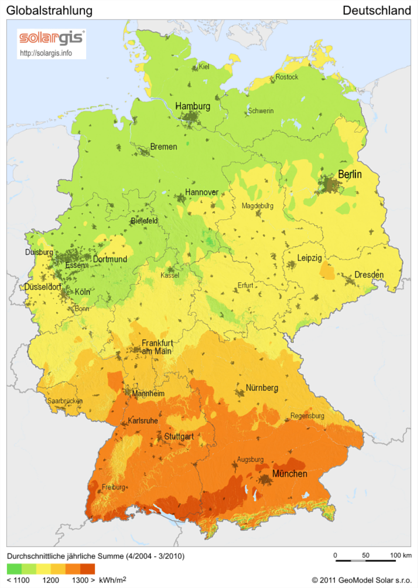
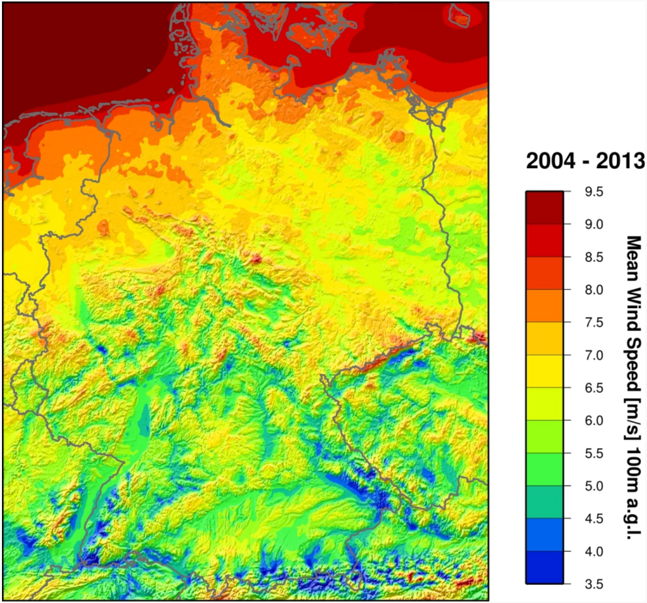
Loose ends from last time

Solar resource distribution in Germany



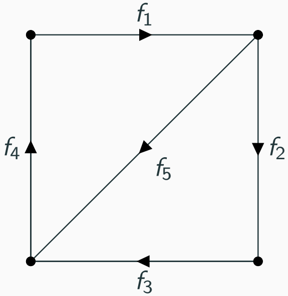
- Solar insolution at top of atmosphere is on average 1361 W/m^2 (orbit is elliptical).
- In Germany average insolation on a horizonal surface is around 1200 kWh/m^2.
- A 1 kW solar panel (around 7 m^2) will generate around 1000 kWh/a.

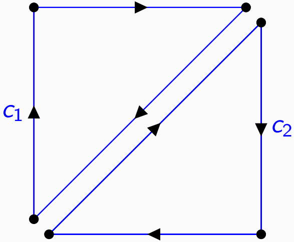
Wind resource distribution in Germany



- Best wind speeds in Germany in North and on hills.
- In theory power output goes like cube $\propto v^3$ of wind speed v.
- In practice power-speed relationship is only partially cubic.

Independent basis of cycles





Two independent cycles:

$c_1 = f_1 + f_5 + f_4$

$c_2 = f_2 + f_3 + -f_5$

The outer cycle is not independent:

$c_3 = f_1 + f_2 + f_3 + f_4 = c_1 + c_2$

(Co)homology analogy

$$K \leftrightarrow \delta \text{ (1d boundary operator)}$$
$$K^t \leftrightarrow d \text{ (0d differential)}$$
$$L = KK^t \leftrightarrow \Delta = d * d \text{ (0d Laplacian)}$$

On a 1d lattice, for each link (difference) from K^t get $u_i - u_{i-1} \sim \frac{d}{dx}$.
From $L = KK^t$ get $2u_i - u_{i-1} - u_{i+1} \sim \frac{d^2}{dx^2}$.

Similarly for 2d lattice.

Full power flow equations

Goal

Last time we said we can (in the linear approximation) express the flow on each line in terms of the voltage angle at the end buses (a relative of $V = IR$) for a line ℓ with reactance x_ℓ as

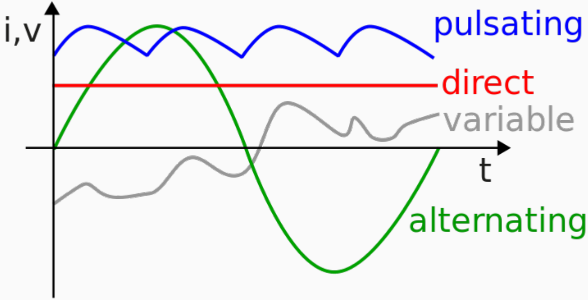
$$f_\ell = \frac{\theta_i - \theta_j}{x_\ell} = \frac{1}{x_\ell} \sum_i K_{i\ell} \theta_i$$

Now we explain where this comes from, and the linear approximation that leads to it.

This is also useful when we consider the synchronisation of oscillators later.

Alternating Current

The majority of electrical power, including what you get out of a wall plug, is transmitted as Alternating Current (AC), i.e. both the voltage and current are sinusoidal waves.



Source: Wikipedia

[Some power is transmitted as Direct Current (DC) under bodies of water and indeed many electronic devices require DC (must convert AC to DC).]

Why alternating current?

Battle of currents! Edison versus Westinghouse/Tesla, etc.

`https://en.wikipedia.org/wiki/War_of_Currents`

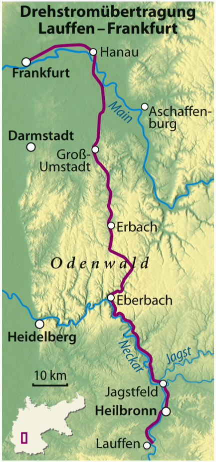
AC won, because it's easy to transform AC to a higher voltage, so you can transmit a given power with a lower current and thus avoid the I^2R resistive losses in power lines.

Reason: $\frac{d}{dt}$ in $\mathcal{E} = \frac{d\Phi}{dt}$; use a solenoid to induce a fluctuating magnetic field in another solenoid with a different number of turns, giving different potential difference.

Frequency of 50 Hz is uniform across Europe (except for train-electricity, e.g. in Germany 16.7 Hz). 60 Hz in USA, half of Japan, etc.

Frankfurt: Home of Long-Distance AC Transmission

First long-distance high-voltage alternating-current transmission in 1891 from hydro plant in Lauffen to Frankfurt for the Elektrotechnische Ausstellung (176 km, 15 kV).



Source: Wikipedia

Sinuisoidal waves

The voltage is usually written in terms of the frequency $\omega = 2\pi f$ and the Root-Mean-Squared (RMS) voltage magnitude V_{rms}

$$V(t) = V_{\mathrm{peak}} \sin(\omega t) = \sqrt{2} V_{\mathrm{rms}} \sin(\omega t)$$

Similarly for the current we have

$$I(t) = I_{\mathrm{peak}} \sin(\omega t - \varphi) = \sqrt{2} I_{\mathrm{rms}} \sin(\omega t - \varphi)$$

Note that they are not necessarily in phase, $\varphi \neq 0$.

The RMS values are useful because then for the average power with $\varphi = 0$ we can forget factors of 2

$$\langle P(t) \rangle = \langle V(t) I(t) \rangle = 2 V_{\mathrm{rms}} I_{\mathrm{rms}} \langle \sin^2(\omega t) \rangle = V_{\mathrm{rms}} I_{\mathrm{rms}}$$

Resistive loads

For purely resistive loads, e.g. a kettle or an electric heater, we have

$$V(t) = RI(t)$$

and thus for a voltage of $V(t) = \sqrt{2}V_{\mathrm{rms}}e^{j\omega t}$ (NB: for engineers $j = \sqrt{-1}$ to avoid confusion with the current i) we have

$$I(t) = \sqrt{2}\frac{V_{\mathrm{rms}}}{R}e^{j\omega t} = \frac{1}{R}V(t)$$

or in terms of the RMS value and phase shift

$$I_{\mathrm{rms}} = \frac{1}{R}V_{\mathrm{rms}}$$
$$\varphi = 0$$

Inductive loads

For purely inductive loads, e.g. a motor during start-up

$$V(t) = L\frac{dI(t)}{dt}$$

and thus for a voltage of $V(t) = \sqrt{2}V_{\mathrm{rms}}e^{j\omega t}$ we get

$$I(t) = \sqrt{2}\frac{V_{\mathrm{rms}}}{j\omega L}e^{j\omega t} = \frac{1}{j\omega L}V(t)$$

or in terms of the RMS value and phase shift

$$I_{\mathrm{rms}} = \frac{1}{\omega L}V_{\mathrm{rms}}$$
$$\varphi = \frac{\pi}{2}$$

We write $X_L = \omega L$ for the inductive reactance, in analogy to the resistance.

Capacitive loads

For purely capacitive loads we have

$$C\frac{dV(t)}{dt} = I(t)$$

and thus for a voltage of $V(t) = \sqrt{2}V_{\mathrm{rms}}e^{j\omega t}$ we get

$$I(t) = \sqrt{2}j\omega C V_{\mathrm{rms}}e^{j\omega t} = j\omega C V(t)$$

or in terms of the RMS value and phase shift

$$I_{\mathrm{rms}} = \omega C V_{\mathrm{rms}}$$
$$\varphi = -\frac{\pi}{2}$$

We write $X_C = \frac{1}{\omega C}$ for the capacitive reactance.

General loads

General loads will have a combination of resistive, capacitive and inductive parts. For an RLC circuit in series the voltage across the components is additive

$$V(t) = RI(t) + L\frac{dI(t)}{dt} + \frac{1}{C}\int_{-infty}^{t} I(\tau)d\tau$$

and therefore for a sinuisoidal voltage with angular frequency ω we get

$$V(t) = \left[R + j\omega L + \frac{1}{j\omega C}\right] I(t)$$

which leads us to define a general complex notion of resistance called impedance

$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j(X_L - X_C) = R + jX$$

where X is the reactance $X = X_L - X_C$.

Impedances and admittances

Thus for a regular sinuisodal setup we have

$$V(t) = ZI(t)$$

where the complex impedance takes care both of the relation of the RMS values of the current and the voltage, and their phase difference. We can decompose Z into real resistance R and real reactance X

$$Z = R + jX$$

The inverse impedance, called the admittance is given by

$$Y = \frac{1}{Z}$$

so that

$$I(t) = YV(t)$$

We can also decompose this into real conductance G and real susceptance B

$$Y = G + jB$$

Simple transmission line

A simple model for a transmission line ℓ between nodes i and j is a resistance R in series with an (inductive) reactance X.

[Typical values are for a 380 kV overhead transmission line e.g. $R = 0.03$ Ohm/km and $X = 0.3$ Ohm/km.]

The voltage at each node (compared to ground) is given by $V_i(t) = \sqrt{2}V_i e^{j(\omega t + \theta_i)}$ where θ_i is the phase offset for each node and V_i is the RMS voltage magnitude.

Now the current in the transmission line is given by

$$I(t) = \frac{1}{R + jX}\left[V_j(t) - V_i(t)\right] = \frac{1}{R + jX}\sqrt{2}V_i e^{i(\omega t + \theta_i)}\left[\frac{V_j}{V_i}e^{j(\theta_j - \theta_i)} - 1\right]$$

Active versus reactive power

Now let's consider the power injection at the first node. This is simply the voltage there multiplied by the current in the transmission line.

It's convenient to eliminate the time-dependent part $e^{i\omega t}$ by multiplying the voltage with the complex conjugate of the current

$$S = P + jQ = \frac{1}{2} V(t) I^*(t)$$

For a resistive load with $V(t) = RI(t)$ this reproduces the active power P.

For loads where the $I(t)$ is not in phase with the voltage, we get a flow of reactive power Q.

$S = P + jQ$ is called the apparent power.

Linearisation: Assumption 1/3

Now if we consider the power injected at the first node we get

$$P_i + jQ_i = \frac{1}{R + jX} V_i^2 \left[\frac{V_j}{V_i} e^{j(\theta_i - \theta_j)} - 1 \right]$$

This is the full non-linear equation for the power flow. Now let's linearise by making some simplifying assumptions.

1. Assume the voltage magnitudes are the same everywhere in the network $V_i = V_j$

$$P_i + jQ_i = \frac{1}{R + jX} V_i^2 \left[e^{j(\theta_i - \theta_j)} - 1 \right]$$

This means power flows primarily according to angle differences in this approximation.

Linearisation: Assumption 2/3

2. Now assume that the voltage angle differences across the transmission line are small enough that $\sin(\theta_i - \theta_j) \sim (\theta_i - \theta_j)$

$$
\begin{aligned}
P_i + jQ_i &= \frac{1}{R + jX} V_i^2 \left[e^{j(\theta_i - \theta_j)} - 1 \right] \\
&\sim \frac{1}{R + jX} V_i^2 \left[j(\theta_i - \theta_j) \right]
\end{aligned}
$$

This assumption is usually valid, since for stability reasons, we usually have in the transmission network $(\theta_i - \theta_j) \leq \frac{\pi}{6}$ (30 degrees).

Linearisation: Assumption 3/3

3. Finally we assume $R << X$ so that we can ignore the resistance R

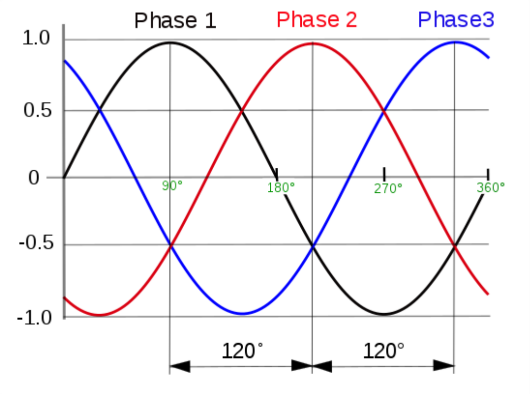
$$
\begin{aligned}
P_i + jQ_i &= \frac{1}{R + jX} V_i^2 \left[j(\theta_i - \theta_j) \right] \\
&\sim \frac{1}{jX} V_i^2 \left[j(\theta_i - \theta_j) \right] \\
&= \frac{V_i^2}{X} (\theta_i - \theta_j)
\end{aligned}
$$

Note that ignoring R means that we ignore resistive losses in the transmission lines and also since $Q_i \sim 0$, we ignore the flow of reactive power. Finally we absorb the voltage into the definition of the per unit reactance $x_\ell = \frac{X}{V_i^2}$ to get

$$
f_\ell = P_i = -P_j = \frac{\theta_i - \theta_j}{x_\ell}
$$

Three-phase power

Electricity is generally generated simultaneously in 3 separate circuits separate by 120 degrees or $\frac{2\pi}{3}$



Source: Wikipedia

In your plug, you only see one phase, but your oven may use all three phases.

Three-phase power

Why three phases? This was settled in the late 1880s.

1. The total power delivery is constant

$$\frac{d}{dt}P(t) = \frac{d}{dt}\left[P_a(t) + P_b(t) + P_c(t)\right] = 0$$

This reduces mechanical stress on generators and motors.

2. The sum of voltages and currents is zero, so no return path required! Saving on materials.

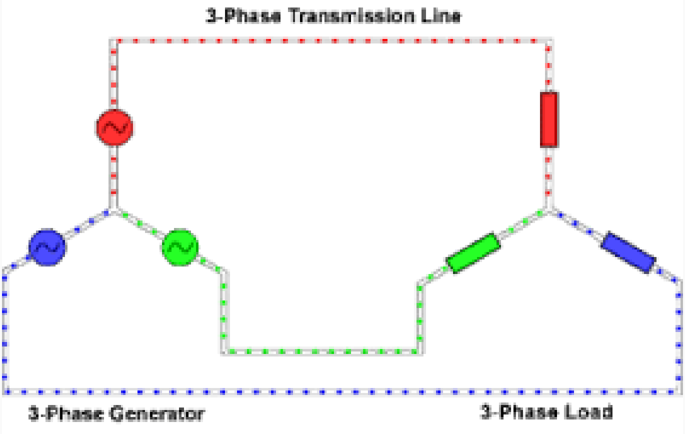
Both facts follow from

$$\sum_{k=0}^{N-1} e^{j\frac{2\pi k}{N}} = 0$$

for $N > 1$.

3. Why $N = 3$ rather than $N = 2$? Allows directional rotating fields for induction motors (thanks Tesla!).

Three-phase power



Source: Wikipedia

Computing the Linear Power Flow

Framing the load flow problem

Suppose we have N nodes labelled by i, and L edges labelled by ℓ forming a directed graph G.

Suppose at each node we have a power imbalance p_i ($p_i > 0$ means its generating more than it consumes and $p_i < 0$ means it is consuming more than it).

Since we cannot create or destroy energy (and we're ignoring losses):

$$\sum_i p_i = 0$$

Question: How do the flows f_ℓ in the network relate to the nodal power imbalances?

Answer: According to the impedances (generalisation of resistance for oscillating voltage/current) and the corresponding voltages.

Kirchhoff's Current Law (KCL)

KCL says (in this linear setting) that the nodal power imbalance at node i is equal to the sum of direct flows arriving at the node. This can be expressed compactly with the incidence matrix

$$p_i = \sum_{\ell} K_{i\ell} f_\ell \qquad \forall i$$

Kirchhoff's Voltage Law (KVL)

KVL says that the sum of voltage differences across edges for any closed cycle must add up to zero.

If the voltage at any node is given by θ_i (this is infact the voltage angle - more next week) then the voltage difference across edge ℓ is

$$\sum_i K_{i\ell}\theta_i$$

And Kirchhoff's law can be expressed using the cycle matrix encoding of independent cycles

$$\sum_\ell C_{\ell c} \sum_i K_{i\ell}\theta_i = 0 \qquad\qquad \forall c$$

[Automatic, since we already said $KC = 0$.]

Kirchhoff's Voltage Law (KVL)

If we express the flow on each line in terms of the voltage angle (a relative of $V = IR$) then for a line ℓ with reactance x_ℓ

$$f_\ell = \frac{\theta_i - \theta_j}{x_\ell} = \frac{1}{x_\ell} \sum_i K_{i\ell} \theta_i$$

KVL now becomes

$$\sum_\ell C_{\ell c} x_\ell f_\ell = 0 \qquad\qquad \forall c$$

Solving the equations

If we combine

$$f_\ell = \frac{1}{x_\ell} \sum_i K_{i\ell}\theta_i$$

with Kirchhoff's Current Law we get

$$p_i = \sum_\ell K_{i\ell} f_\ell = \sum_\ell K_{i\ell} \frac{1}{x_\ell} \sum_j K_{j\ell}\theta_j$$

This is a weighted Laplacian. If we write $B_{k\ell}$ for the diagonal matrix with $B_{\ell\ell} = \frac{1}{x_\ell}$ then

$$L = KBK^t$$

and we get a discrete Poisson equation for the θ_i sourced by the p_i

$$p_i = \sum_j L_{ij}\theta_j$$

We can solve this for the θ_i and thus find the flows.

Solving the equations

Given p_i at every node, we want to find the flows f_ℓ. We have the equations

$$p_i = \sum_j L_{ij}\theta_j$$
$$f_\ell = \frac{1}{x_\ell}\sum_i K_{i\ell}\theta_i$$

Basic idea: invert L to get θ_i in terms of p_i

$$\theta_i = \sum_k (L^{-1})_{ik} p_k$$

then insert to get the flows as a linear function of the power injections p_i

$$f_\ell = \frac{1}{x_\ell}\sum_{i,k} K_{i\ell}(L^{-1})_{ik} p_k = \sum_k \mathrm{PTDF}_{\ell k} p_k$$

Inverting Laplacian L

There is one small catch: L is not invertible since we saw last time it has (for a connected network) one zero eigenvalue, with eigenvector $(1, 1, \ldots 1)$, since by construction $\sum_j L_{ij} = 0$.

This is related to a gauge freedom to add a constant to all voltage angles

$$\theta_i \to \theta_i + c$$

which does not affect physical quantities:

$$p_i = \sum_j L_{ij}(\theta_j + c) = \sum_j L_{ij}(\theta_j)$$
$$f_\ell = \frac{1}{x_\ell} \sum_i K_{i\ell}(\theta_i + c) = \frac{1}{x_\ell} \sum_i K_{i\ell}(\theta_i)$$

Typically choose a slack or reference bus such that $\theta_0 = 0$.

Inverting Laplacian L

Two solutions:

1. Set $\theta_0 = 0$, invert the lower-right $(N-1) \times (N-1)$ part of L to find the remaining $\{p_i\}_{i=1,\dots N-1}$ in terms of the $\{\theta_i\}_{i=1,\dots N-1}$, then derive p_0 from $\sum_i p_i = 0$.
2. Use the Moore-Penrose pseudo-inverse.

Write L in terms of its basis of orthonormal eigenvectors

$$L = \sum_n |\Phi_n\rangle \, \lambda_n \, \langle\Phi_n|$$

then the Moore-Penrose pseudo-inverse is:

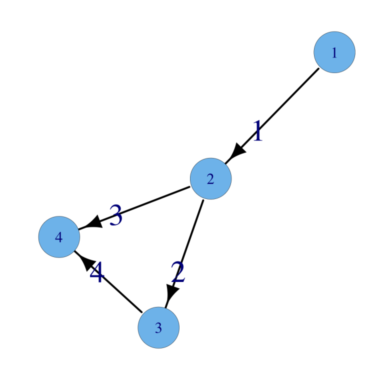
$$L^\dagger = \sum_{n|\lambda_n \neq 0} \frac{|\Phi_n\rangle \langle\Phi_n|}{\lambda_n}$$

4-node example

$$\mathbf{K}_{i\ell} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix}$$

$$\mathbf{L}_{ij} = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix}$$

$$\mathbf{PTDF}_{\ell i} = \begin{pmatrix} 0 & -1 & -1 & -1 \\ 0 & 0 & -2/3 & -1/3 \\ 0 & 0 & -1/3 & -2/3 \\ 0 & 0 & 1/3 & -1/3 \end{pmatrix}$$

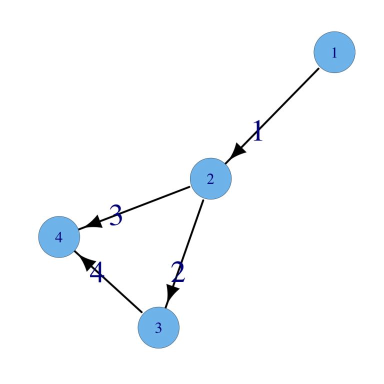


PTDF as sensitivity

Can also ‘experimentally’ determine the Power Transfer Distribution Factors (PTDF) by choosing a slack bus (in this case bus 1).

Each column (labelled by i) is then the resulting line flows if we have a simple power transfer from bus i to the slack $p_i = 1$ and $p_1 = -1$.

$$\mathbf{PTDF}_{\ell i} = \begin{pmatrix} 0 & -1 & -1 & -1 \\ 0 & 0 & -2/3 & -1/3 \\ 0 & 0 & -1/3 & -2/3 \\ 0 & 0 & 1/3 & -1/3 \end{pmatrix}$$



Consequences of limiting power transfers

Thermal limits

You cannot pass infinite current through a transmission line.

As it warms, it sags, then it will hit a building/tree and cause a short-circuit. [Or you may get voltage instability.]

Typically each line has a well-defined thermal limit on the amount of current that can flow through it.

$$|f_\ell| \leq F_\ell$$

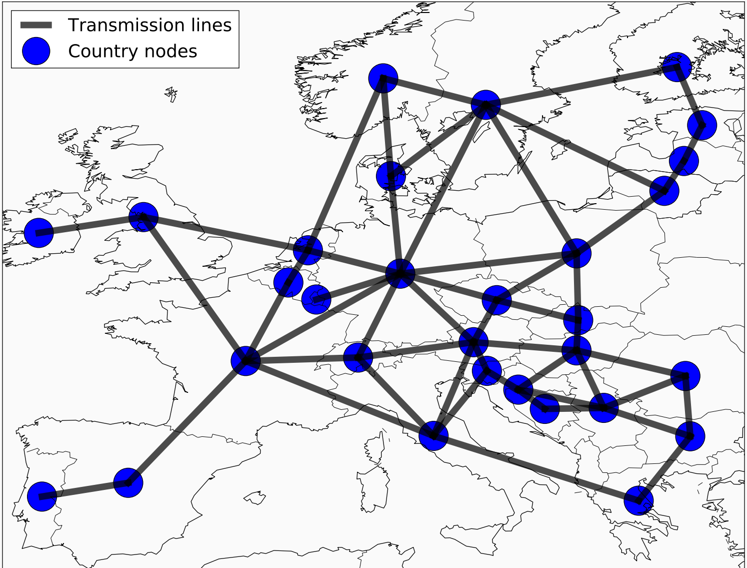
These limits prevent the transfer of renewable energy.

Germany curtailment example

See https://pypsa.org/examples/scigrid-lopf-then-pf.html.

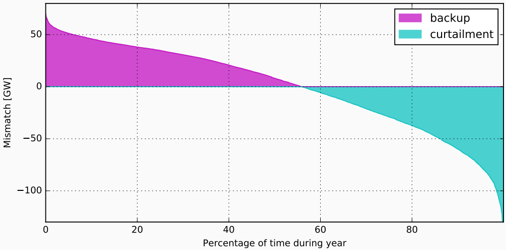
European transmission versus backup energy

Consider backup energy in a simplified European grid:

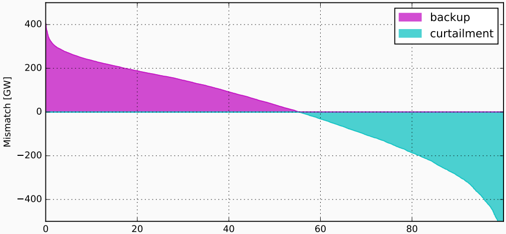


DE versus EU backup energy from last time

Germany needed backup generation for 31% of total load:



Europe needed Backup generation for only 24% of the total load:



European transmission versus backup energy

Transmission needs across a fully renewable European power system by Rodriguez, Becker, Andresen, Heide, Greiner, Renewable Energy, 2014